



Class 11 Maths || Chapter 9 : Sequence and Series || Class Notes

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Class-11-Maths Chapter [9]

Sequences and Series

Sequence

term
↓ ↓ ↓
1, 3, 5, 7, 9, 11

→ Collection of particular numbers in particular pattern.

Series

$= 1 + 3 + 5 + 7 + 9 + 11$ → representation of numbers of sequence with '+' sign

Sum of Series = 36

Finite Sequence → no. of terms = Finite (1, 2, 3, 4, ..., 100)

Infinite Sequence → no. of terms = infinite (1, 2, 3, ...)

Sequence → $a_1, a_2, a_3, a_4, \dots, a_n, a_{n+1}, a_{n+2}, \dots$

Series → $a_1 + a_2 + a_3 + \dots + (a_n) + (a_{n+1}) + (a_{n+2}) + \dots + a_{1000}$



$$\sum_{r=1}^{1000} a_r$$

nth term → General Term = Formula

YouTube पर इस Chapter के FREE Videos देखिए (Playlist की Link नीचे है)

https://www.youtube.com/watch?v=2R_dTnCO6N4&list=PLPFrnOppwwkTM3uOtkLgG6oJ0cW7G_tua

Exercise [9.1]

[Q.1] $a_n = n \cdot (n+2)$

$$a_1 = 1 \cdot (1+2) = 1 \times 3 = 3$$

$$a_2 = 2 \cdot (2+2) = 2 \times 4 = 8$$

$$a_3 = 3 \cdot (3+2) = 15$$

$$a_4 = 4 \cdot (4+2) = 24$$

$$a_5 = 5 \cdot (5+2) = 35$$

$$\underline{3, 8, 15, 24, 35, \dots}$$

[Q.2] $a_n = \frac{n}{n+1}$

$$a_1 = \frac{1}{1+1} = \frac{1}{2}$$

$$a_2 = \frac{2}{2+1} = \frac{2}{3}$$

$$a_3 = \frac{3}{3+1} = \frac{3}{4}$$

$$a_4 = \frac{4}{4+1} = \frac{4}{5}$$

$$a_5 = \frac{5}{5+1} = \frac{5}{6}$$

[Q.3] $a_n = 2^n$

$$a_1 = 2^1 = 2$$

$$a_2 = 2^2 = 4$$

$$a_3 = 2^3 = 8$$

$$a_4 = 2^4 = 16$$

$$a_5 = 2^5 = 32$$

[Q.4] $a_n = \frac{2n-3}{6}$

$$a_1 = \frac{2 \times 1 - 3}{6} = \frac{2-3}{6} = \frac{-1}{6}$$

$$a_2 = \frac{2 \times 2 - 3}{6} = \frac{4-3}{6} = \frac{1}{6}$$

$$a_3 = \frac{2 \times 3 - 3}{6} = \frac{3}{6} = \frac{1}{2}$$

$$a_4 = \frac{2 \times 4 - 3}{6} = \frac{5}{6}$$

$$a_5 = \frac{2 \times 5 - 3}{6} = \frac{7}{6}$$

[Q.5] $a_n = (-1)^{\downarrow n-1} \cdot 5^{\downarrow n+1}$

$$a_1 = (-1)^{1-1} \cdot 5^{1+1} = (-1)^0 \cdot 5^2 = 1 \times 25 = 25$$

$$a_2 = (-1)^{2-1} \cdot 5^{2+1} = -125$$

$$a_3 = (-1)^{3-1} \cdot 5^{3+1} = +625$$

$$a_4 = (-1)^{4-1} \cdot 5^{4+1} = -3125$$

$$a_5 = (-1)^{5-1} \cdot 5^{5+1} = +15625$$

[Q.6] $a_n = n \cdot \left(\frac{n^2+5}{4} \right)$

$$a_1 = 1 \cdot \left(\frac{1^2+5}{4} \right) = \frac{3}{2}$$

$$a_2 = 2 \cdot \left(\frac{2^2+5}{4} \right) = \frac{9}{2}$$

$$a_3 = 3 \cdot \left(\frac{3^2+5}{4} \right) = \frac{21}{2}$$

$$a_4 = 4 \cdot \left(\frac{4^2+5}{4} \right) = 21$$

$$\begin{aligned} a_5 &= 5 \cdot \left(\frac{5^2+5}{4} \right) \\ &= \frac{75}{2} \end{aligned}$$

Q.7

$$a_n = 4n - 3$$

$$a_{17} = 4(17) - 3 = 68 - 3 = 65 \checkmark$$

$$a_{24} = 4(24) - 3 = 96 - 3 = 93 \checkmark$$

Q.8.

$$a_n = \frac{n^2}{2^n}$$

$$a_7 = \frac{(7^2)}{2^7} = \frac{49}{128} \checkmark$$

Q.9

$$a_n = (-1)^{n-1} \cdot n^3$$

$$\begin{aligned} \checkmark a_9 &= (-1)^{9-1} \cdot 9^3 \\ &= (-1)^8 \cdot 729 \\ &= +729 \checkmark \end{aligned}$$

Q.10

$$a_n = \frac{n(n-2)}{n+3}$$

$$\begin{aligned} \checkmark a_{20} &= \frac{20 \times (20-2)}{20+3} \\ &= \frac{20 \times 18}{23} = \frac{360}{23} \checkmark \end{aligned}$$

Q.11

$$a_1 = 3, \quad a_n = 3(a_{n-1}) + 2, \quad n > 1$$

$$a_n = 3 \cdot (a_{n-1}) + 2$$

n=2

$$a_2 = 3 \times a_{2-1} + 2 = 3 \times (a_1) + 2 = 3 \times 3 + 2 = 11$$

n=3

$$a_3 = 3 \cdot (a_{3-1}) + 2 = 3 \times (a_2) + 2 = 3 \times 11 + 2 = 35$$

$$a_1 = 3$$

$$a_2 = 11$$

$$a_3 = 35$$

n=4

$$\begin{aligned} a_4 &= 3 \cdot a_3 + 2 \\ &= 3 \times 35 + 2 = 107 \end{aligned}$$

$$a_5 = 3 \times a_4 + 2$$

$$= 3 \times 107 + 2 = 323$$

Series:

$$3 + 11 + 35 + 107 + 323 + \dots$$

Q.12 $a_1 = -1$

$$a_n = \frac{a_{n-1}}{n}$$

$n=2$ $a_2 = \frac{a_{2-1}}{2} = \frac{a_1}{2} = \frac{-1}{2}$

$$a_3 = \frac{a_2}{3} = \frac{\left(-\frac{1}{2}\right)}{3} = -\frac{1}{6}$$

$$a_4 = \frac{a_3}{4} = \frac{\left(-\frac{1}{6}\right)}{4} = -\frac{1}{24}$$

$$a_5 = \frac{a_4}{5} = \frac{\left(-\frac{1}{24}\right)}{5} = -\frac{1}{120}$$

$$\text{Series} = (-1) + \left(-\frac{1}{2}\right) + \left(-\frac{1}{6}\right) + \left(-\frac{1}{24}\right) + \left(-\frac{1}{120}\right) + \dots$$

Q.13 $a_1 = a_2 = 2$

$$a_n = a_{n-1} - 1, \quad n > 2$$

$n=3$ $a_3 = a_{3-1} - 1 = a_2 - 1 = 2 - 1 = 1$

$n=4$ $a_4 = a_3 - 1 = 1 - 1 = 0$

$n=5$ $a_5 = a_4 - 1 = 0 - 1 = -1$

$$\text{Series} = 2 + 2 + 1 + 0 + (-1) + \dots$$

Q.14 Fibonacci Sequence $a_1 = a_2 = 1$

$$a_n = a_{n-1} + a_{n-2}, \quad n > 2$$

$$\underline{n=3}, \quad a_3 = a_2 + a_1 = 1 + 1 = 2$$

$$n=4, \quad a_4 = a_3 + a_2 = 2 + 1 = 3$$

$$n=5, \quad a_5 = a_4 + a_3 = 3 + 2 = 5$$

$$n=6, \quad a_6 = a_5 + a_4 = 5 + 3 = 8$$

$$\frac{a_{n+1}}{a_n} = ? \quad n = \underline{1, 2, 3, 4, 5}$$

$$n=1, \quad \frac{a_2}{a_1} = \frac{1}{1} = 1 \quad \checkmark$$

$$n=2, \quad \frac{a_3}{a_2} = \frac{2}{1} = 2 \quad \checkmark$$

$$n=3, \quad \frac{a_4}{a_3} = \frac{3}{2} \quad \checkmark$$

$$n=4, \quad \frac{a_5}{a_4} = \frac{5}{3} \quad \checkmark$$

$$n=5, \quad \frac{a_6}{a_5} = \frac{8}{5} \quad \checkmark$$

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Class - 11 - Maths. Chapter (9)

Arithmetic Progression (A.P.) $\Rightarrow$

$$\boxed{a_n = a_{n-1} + d} \rightarrow \text{Fixed no.}$$

Common Difference
(C.D.)

Let First term = a

$$CD = d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

$$AP \rightarrow a, a+d, a+2d, a+3d, \dots, \frac{a+(n-1)d}{1}$$

$$\begin{array}{ccccccc} & \uparrow & \uparrow & \uparrow & \uparrow & & \\ a_1 & a_2 & a_3 & a_4 & & & \\ (1) & & & & & & \end{array}$$

General $(a_n) = n^{\text{th}}$ term

General Term $a_n = n^{\text{th}} \text{ term} = a + (n-1)d$

$$\text{Sum of AP.} = S_n = a_1 + a_2 + a_3 + \dots + a_n = \frac{n}{2} (2a + (n-1)d)$$

$$S_n = \frac{n}{2} (a + \underset{\substack{\uparrow \\ \text{last} \\ \text{term}}}{l}) = \frac{n}{2} (a_1 + a_n)$$

$$a_n = (S_n) - (S_{n-1})$$

Properties:

① If $\underbrace{a, b, c}_{b-a = c-b}$ are in AP, then $b = \frac{a+c}{2}$ or $2b = a+c$

②

AP

$+K$ $-K$ $\times K$ $\div K$

constant (for each term) of A.P.

new sequence
new Resultant sequence

New AP.

$$\begin{array}{ccccc} AP_1 & + & AP_2 & & = AP_3 \\ \uparrow & & \uparrow & & \swarrow \\ a_1 + b_1 & = & c_1 & & \\ a_2 + b_2 & = & c_2 & & \end{array}$$

$$\checkmark \quad x \quad \checkmark \quad \underline{a-3d} \quad \checkmark \quad a-2d \quad \checkmark \quad \underline{a-d} \quad \cancel{a} \quad \checkmark \quad a+d \quad \checkmark \quad a+2d \quad \checkmark \quad \underline{a+3d} \quad x$$

③ Assumption of terms in AP. (Condition $\rightarrow$ their Sum is given).

Assume 3 - numbers $a-d, a, a+d$ $(C=D=d)$

Assume 5-numbers

$$CD = d \quad \underbrace{a-2d}, \underbrace{a-d}, \underbrace{a}, \underbrace{a+d}, \underbrace{a+2d}$$

Assume 4-numbers

Assume 4-numbers
 $cd = 2d$ $a-3d, a-d, a+d, a+3d$

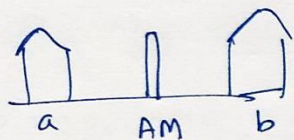
Assume 6-numbers

Assume 6 numbers $C.D = 2d$

$a-5d, a-3d, a-d, a+d, a+3d, a+5d$

Arithmetic Mean (AM)

① AM of two no. a & $b = \frac{a+b}{2}$



$$AM = \frac{a+b}{2}$$

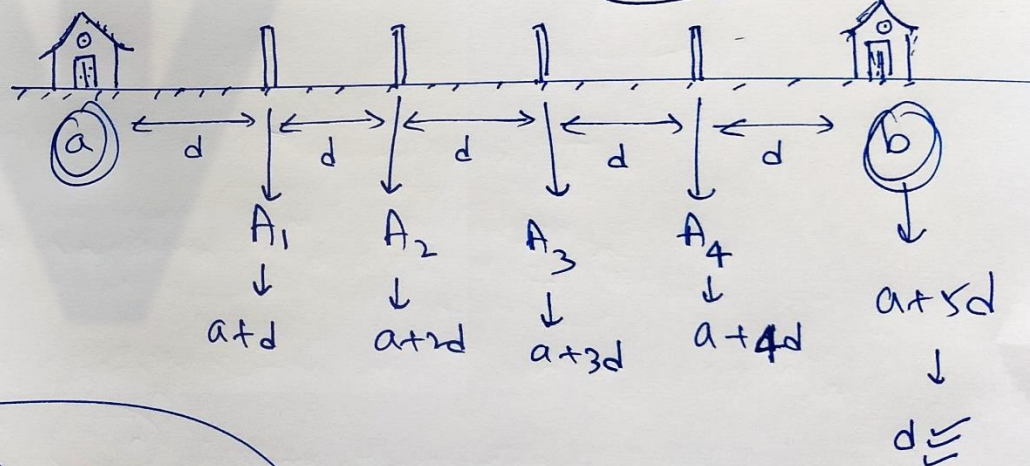
AM of a, b, c, d
 $AM = \frac{a+b+c+d}{4}$

② n -AMs $\rightarrow$ find $\rightarrow$ between (a) & (b)

$A_1, A_2, A_3, \dots, A_n$

So that

$a, A_1, A_2, A_3, \dots, A_n, b$ form AP



only one unknown = d

$$\begin{cases} A_1 = a+d \\ A_2 = a+2d \\ \vdots \\ A_n = a+n \cdot d \end{cases}$$

Hint

$$b = a + (n+1) \cdot d$$

$$\begin{aligned} b &= a + (n+1) \cdot d \\ b - a &= (n+1) \cdot d \\ \Rightarrow \boxed{\frac{b-a}{n+1} = d} \end{aligned}$$

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Class-11-Maths

Exercise 9.2 → A.P.

① n^{th} term = $a_n = a + (n-1) \cdot d \leftarrow \text{d = C.D.}$

② $a_n = S_n - S_{n-1}$

③ $S_n = \frac{n}{2} (2a + (n-1) \cdot d) = a_1 + a_2 + \dots + a_n$

④ $S_n = \frac{n}{2} (a + l) = \frac{n}{2} (a_1 + a_n)$

Ex. 9.2

Q.1 Sum of odd integers from '1' to '2001'

$S_n = 1 + 3 + 5 + 7 + \dots + 1999 + 2001$
 $\uparrow \qquad \qquad \qquad \uparrow$
 $n=? \qquad a_1 = a \qquad l = a_n = a + (n-1) \cdot d$
 $d = \text{Common Difference} = 2$

$$a_n = a + (n-1) \cdot d$$

$$\Rightarrow 2001 = 1 + (n-1) \cdot 2$$

$$\Rightarrow 2000 = (n-1) \cdot 2$$

$$\Rightarrow 1000 = n-1$$

$$\Rightarrow \text{1001} = n$$

$$S_n = S_{1001} = \frac{n}{2} (a + l)$$

$$S_{1001} = \frac{1001}{2} (1 + 2001)$$

$$= \frac{1001}{2} \times \cancel{2002}^{1001}$$

$$= 1001 \times 1001$$

$$= 1002001 \quad \checkmark$$

Q.2 Sum of all natural no. lying (b/w) 100 & 1000
which are multiple of '5'

$$S_n = 105 + 110 + 115 + \dots + 990 + 995$$

$\uparrow \quad \quad \uparrow$
 $a_1 = a \quad a_2$
 $\quad \quad \quad \uparrow \quad \quad \uparrow$
 $\quad \quad \quad a_{n-1} \quad a_n = 995$

$d = 5$

$$S_n = \frac{n}{2} (a + l)$$

$$= \frac{n}{2} (105 + 995)$$

$$= \frac{179}{2} (1100)$$

$$= \frac{196900}{2}$$

$$= 98450 \checkmark$$

$$\Rightarrow a + (n-1) \cdot d = 995$$

$$\Rightarrow 105 + (n-1) \cdot 5 = 995$$

$$\Rightarrow (n-1) \cdot 5 = 890$$

178

$$\Rightarrow n-1 = 178$$

$n = 179$

$$\begin{array}{r} 179 \\ \times 11 \\ \hline 179 \\ 179 \\ \hline 196900 \end{array}$$

Q.3

$$a_1 = a = 2$$

$$a_{20} = ? = -112$$

$$a_1, a_2, a_3, a_4, a_5$$

$$a_6, a_7, a_8, a_9, a_{10}$$

ATQ.

$$(a_1 + a_2 + a_3 + a_4 + a_5) = \frac{1}{4} (a_6 + a_7 + a_8 + a_9 + a_{10})$$

$$\Rightarrow (a + a + d + a + 2d + a + 3d + a + 4d) = \frac{1}{4} (a + 5d + a + 6d + a + 7d + a + 8d + a + 9d)$$

$$\Rightarrow \frac{a + 2d}{(5a + 10d)} = \frac{1}{4} \frac{a + 7d}{(5a + 35d)}$$

$$\Rightarrow 4a + 8d = a + 7d$$

$$\Rightarrow 3a + d = 0$$

$$\Rightarrow d = -3a$$

$a = 2$

$$d = -3(2)$$

$$d = -6$$

$$a_{20} = a + 19d$$

$$= 2 + 19(-6)$$

$$= 2 + (-114)$$

$$a_{20} = -112$$

Q.4

$$-6, -\frac{11}{2}, -5, \dots$$

$\downarrow$
 $\textcircled{a}$

$$\text{Sum} = -25$$

How many Terms = ? = n

Let the no. of terms = n

$$S_n = \text{Sum} = -25$$

$$\Rightarrow \frac{n}{2} (2a + (n-1) \cdot d) = -25$$

$$\Rightarrow n \left[2 \times (-6) + (n-1) \cdot \frac{1}{2} \right] = -25 \times 2$$

$$\Rightarrow n \left(-12 + \frac{n-1}{2} \right) = -50$$

$$\Rightarrow n \left(\frac{-24 + n-1}{2} \right) = -50$$

$$\Rightarrow n(n-25) = -100$$

$$= \text{AP. } d = a_2 - a_1$$

$$d = \left(-\frac{11}{2}\right) - (-6)$$

$$= -\frac{11}{2} + 6$$

$$= \frac{-11+12}{2} = \frac{1}{2}$$

$$n^2 - 25n + 100 = 0$$

$$\Rightarrow n^2 - 20n - 5n + 100 = 0$$

$$\Rightarrow n(n-20) - 5(n-20) = 0$$

$$\Rightarrow (n-20)(n-5) = 0$$

$$\downarrow \quad \downarrow$$
$$\textcircled{n=20} \quad \textcircled{n=5} \quad \checkmark$$

$$\underbrace{a_1 \ a_2 \ a_3 \ a_4 \ a_5}_{-25} \quad \underbrace{a_6 \ a_7 \ \dots \ a_{20}}_0$$
$$\underbrace{\hspace{10em}}_{-25}$$

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Class-11- Maths Exercise (9.2)

✓ n^{th} term $= a_n = a + (n-1) \cdot d$

✓ $a_n = S_n - S_{n-1}$

✓ $S_n = \frac{n}{2} (2a + (n-1) \cdot d)$

$a = a_1$

$d = \text{C.D.}$

✓ $S_n = \frac{n}{2} (a + l) = \frac{n}{2} (a_1 + a_n)$

Q.6

$25, 22, 19, \dots, a_n$ $a = a_1 = 25$
 $d = -3$

$S_n = 116$

no. of terms = n

let last term $= a_n$

$S_n = 116$

$\Rightarrow \frac{n}{2} (2a + (n-1) \cdot d) = 116$

$\Rightarrow n (2 \times 25 + (n-1) (-3)) = 232$

$\Rightarrow n (50 - 3n + 3) = 232$

$\Rightarrow n (53 - 3n) = 232$

$\Rightarrow -3n^2 + 53n = 232$

$\Rightarrow 3n^2 - 53n + 232 = 0$

$\Rightarrow$ $\begin{matrix} \swarrow & \searrow \\ \boxed{-24} & \times & \boxed{-29} \end{matrix}$ $\rightarrow 3 \times 232$

$= 3 \times 2 \times 116$

$= 3 \times 2 \times 2 \times 58$

$= 3 \times 2 \times 2 \times 2 \times 29$

$= 24 \times 29$

$\Rightarrow 3n^2 - 24n - 29n + 232 = 0$

$\Rightarrow 3n(n-8) - 29(n-8) = 0$

$\Rightarrow (n-8)(3n-29) = 0$

$\downarrow$

$\downarrow$

$n = 8 \in \mathbb{N}$

$n = \frac{29}{3} \notin \mathbb{N}$

✓

X

$$a_n = a + (n-1) \cdot d$$

$$\begin{aligned} \text{Last } a_8 &= 25 + (8-1) \cdot (-3) \\ &= 25 + 7 \cdot (-3) \\ &= 25 - 21 \end{aligned}$$

$$a_8 = 4$$

Q.7 k^{th} term $= a_k = 5k+1$

$$\text{AP} \rightarrow a_1, a_2, a_3, a_4, \dots$$

$$(k=1) \quad a_1 = 5+1=6 \rightarrow a=6$$

$$(k=2) \quad a_2 = 5 \times 2 + 1 = 11$$

$$d = 11 - 6 = 5$$

$$(k=3) \quad a_3 = 5 \times 3 + 1 = 16$$

$$S_n = \frac{n}{2} (2a + (n-1) \cdot d) = \frac{n}{2} [2 \times 6 + (n-1) 5]$$

$$S_n = \frac{n}{2} (12 + 5n - 5) = \frac{n}{2} (5n + 7) = \frac{5n^2}{2} + \frac{7n}{2}$$

Q.8 $S_n = p \cdot n + q \cdot n^2$ $p, q = \text{constants}$

$$\text{Common Difference} = ? = d = a_2 - a_1$$

$$S_n = p \cdot n + q \cdot n^2 = a_1 + a_2 + \dots + a_n$$

$$S_1 = a_1 = p + q \quad \text{--- (1)}$$

$$S_2 = a_1 + a_2 = 2p + 4q \quad \text{--- (2)}$$

By eqⁿ (1) & (2): $\text{eqⁿ (2)} - \text{eqⁿ (1)}$

$$a_2 = p + 3q$$

$$\text{C.D.} = d = a_2 - a_1$$

$$d = (p + 3q) - (p + q)$$

$$d = 2q$$

Note: $d = 2 \times \text{coeff. of } n^2$

$$(S_n)$$

Q.9 2 A.P.s $\begin{cases} \rightarrow a_1, a_2, a_3, \dots \rightarrow \text{First term} = a, \text{CD} = d \text{ (let)} \rightarrow \text{Sum} = S_n^I \\ \rightarrow b_1, b_2, b_3, \dots \rightarrow \text{First term} = b, \text{CD} = e \text{ (let)} \rightarrow \text{Sum} = S_n^{II} \end{cases}$
 ratio of sum of n -terms $= (5n+4) : (9n+6)$

ratio of their 18th terms = ?

$$\Rightarrow \frac{a_{18}}{b_{18}} = \frac{a + 17 \cdot d}{b + 17 \cdot e} = ?$$

Given, ratio of sum of n -terms $= \frac{5n+4}{9n+6}$

$$\Rightarrow \frac{S_n^I}{S_n^{II}} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{\frac{n}{2} [2a + (n-1) \cdot d] / 2}{\frac{n}{2} [2b + (n-1) \cdot e] / 2} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{a + \left(\frac{n-1}{2}\right) \cdot d}{b + \left(\frac{n-1}{2}\right) \cdot e} = \frac{5n+4}{9n+6} \quad \text{--- (1)}$$

$$\text{let } \left(\frac{n-1}{2}\right) = 17$$

$$\Rightarrow n-1 = 34$$

$$\Rightarrow \boxed{n=35}$$

put $n=35$ in eqⁿ (1)

$$\Rightarrow \frac{a + 17 \cdot d}{b + 17 \cdot e} = \frac{5 \times 35 + 4}{9 \times 35 + 6}$$

$$\Rightarrow \frac{a_{18}}{b_{18}} = \frac{175 + 4}{315 + 6}$$

$$= \frac{179}{321}$$

$$\boxed{a_{18} : b_{18} = 179 : 321}$$

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Class-11 Maths: Exercise 9.2

Q.10 $S_p = S_q$

$$S_n = \frac{n}{2} (2a + (n-1) \cdot d)$$

$$\Rightarrow \frac{p}{2} [2a + (p-1) \cdot d] = \frac{q}{2} [2a + (q-1) \cdot d]$$

? ?
let let

$$\Rightarrow \underline{2a}p + \underline{(p-1) \cdot p d} = \underline{2a}q + \underline{(q-1) \cdot q d}$$

$$\Rightarrow 2a(p-q) + d[p^2 - p - q^2 + q] = 0$$

$$\Rightarrow \underline{2a(p-q)} + \underline{d \cdot [(p-q) \cdot (p+q) - (p-q)]} = 0$$

Sum of first $(p+q)$ terms $= S_{p+q} = \frac{p+q}{2} [2a + (p+q-1) \cdot d]$

0

$$S_{p+q} = \left(\frac{p+q}{2} \right) (0)$$

$$= 0 \quad \checkmark$$

$$\Rightarrow \underline{2a \cdot (p-q)} + \underline{d \cdot (p-q) \cdot [p+q-1]} = 0$$

$$\Rightarrow \underbrace{(p-q)}_{\substack{p \neq q \\ \neq 0}} \cdot \underbrace{\{2a + d \cdot [p+q-1]\}}_0 = 0$$

Q.11

$$S_p = a$$

$$S_q = b$$

$$S_r = c$$

first term = A } let
 $CD = D$

$$\begin{matrix} \uparrow \textcircled{Q} \\ \downarrow \textcircled{P} \end{matrix} (q-r)$$

$(A, D) \times$

$$S_p = a$$

$$\Rightarrow \frac{P}{2} (2A + (P-1) \cdot D) = a$$

$$\Rightarrow \frac{1}{2} (2A + (P-1) \cdot D) = \frac{a}{P}$$

$$\Rightarrow \frac{a}{P} = A + \frac{P \cdot D}{2} - \frac{D}{2}$$

$$\Rightarrow \boxed{\frac{a}{P} = A - \frac{D}{2} + \frac{P \cdot D}{2}}$$

$(q-r)$ multiply

$$\Rightarrow \frac{a}{P} (q-r) = \left(A - \frac{D}{2} + \frac{P \cdot D}{2} \right) \cdot (q-r)$$

$$\Rightarrow \boxed{\begin{aligned} \frac{a}{P} (q-r) &= A \cdot q - A \cdot r - \frac{D}{2} \cdot q + \frac{D}{2} \cdot r \\ &\quad + \frac{D}{2} \cdot Pq - \frac{D}{2} \cdot Pr \end{aligned}}$$

$$\frac{a}{P} (q-r) = Aq - Ar - \frac{D}{2} \cdot q + \frac{D}{2} \cdot r + \frac{D}{2} \cdot Pq - \frac{D}{2} \cdot Pr$$

$$\frac{b}{Q} (r-p) = A \cdot r - A \cdot p - \frac{D}{2} \cdot r + \frac{D}{2} \cdot p + \frac{D}{2} \cdot qr - \frac{D}{2} \cdot pq$$

$$\frac{c}{R} (p-q) = A \cdot p - A \cdot q - \frac{D}{2} \cdot p + \frac{D}{2} \cdot q + \frac{D}{2} \cdot pr - \frac{D}{2} \cdot qr$$

+

$$\frac{a}{P} (q-r) + \frac{b}{Q} (r-p) + \frac{c}{R} (p-q) = 0$$

H.P.

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Class-11-Maths Exercise (9.2)

Revision $\left\{ \begin{array}{l} a_n = a + (n-1) \cdot d \\ a_n = S_n - S_{n-1} \\ S_n = \frac{n}{2} (2a + (n-1) \cdot d) \\ S_n = \frac{n}{2} (a + l) \end{array} \right\}$

Q.12 an A.P. $\begin{cases} \rightarrow \text{first term} = a \\ \rightarrow \text{C.D.} = d \end{cases}$ (let)

Ratio of sums of m & n terms $= m^2 : n^2$

To prove $\boxed{a_m : a_n = (2m-1) : (2n-1)}$

$$\frac{S_m}{S_n} = \frac{m^2}{n^2} \quad \rightarrow \quad \frac{a_m}{a_n} = \frac{a + (m-1) \cdot d}{a + (n-1) \cdot d}$$

$$\Rightarrow \frac{\cancel{m} (2a + (m-1) \cdot d)}{\cancel{n} (2a + (n-1) \cdot d)} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{[2a + (m-1) \cdot d] / 2}{[2a + (n-1) \cdot d] / 2} = \frac{m}{n}$$

$$\Rightarrow \frac{a + (\frac{m-1}{2}) \cdot d}{a + (\frac{n-1}{2}) \cdot d} = \frac{m}{n} \quad \text{--- (1)}$$

let $\frac{m-1}{2} \rightarrow (M-1)$ Substitute.

$\frac{n-1}{2} \rightarrow (N-1)$ Substitute.

$$\frac{m-1}{2} = M-1$$

$$m = 2M - 2 + 1$$

$$m = 2M - 1$$

$$n = 2N - 1$$

By substitution in eqⁿ (1):

$$\Rightarrow \frac{a + (M-1) \cdot d}{a + (N-1) \cdot d} = \frac{2M-1}{2N-1}$$

$$\Rightarrow \frac{a_M}{a_N} = \frac{2M-1}{2N-1}$$

$$\begin{array}{l} M \rightarrow m \\ N \rightarrow n \end{array}$$

$$\Rightarrow \frac{a_m}{a_n} = \frac{2m-1}{2n-1} \quad \underline{\underline{\text{H.P.}}}$$

$$(13) \quad S_n = 3n^2 + 5n$$

$$a_m = \text{m}^{\text{th}} \text{ term} = 164,$$

$$m = ?$$

$$S_n = 3n^2 + 5n = a_1 + a_2 + \dots + a_n$$

$$S_1 = \underline{a_1} = 3 + 5 = \underline{8} = \text{first term.}$$

$$S_2 = a_1 + a_2 = 3 \times 4 + 5 \times 2 = 12 + 10 = 22$$

$$\cancel{a_1 + 8} \quad a_1 + a_2 = 22$$

$$8 + a_2 = 22 \Rightarrow \boxed{a_2 = 14}$$

$$a_1 = 8 = a$$

$$a_2 = 14$$

$$d = a_2 - a_1 = 14 - 8 = 6 \quad \checkmark$$

$$\text{m}^{\text{th}} \text{ term} = 164 \quad \Rightarrow m-1 = \frac{156}{6}$$

$$\Rightarrow a_m = a + (m-1) \cdot d = 164$$

$$\Rightarrow 8 + (m-1) \cdot 6 = 164$$

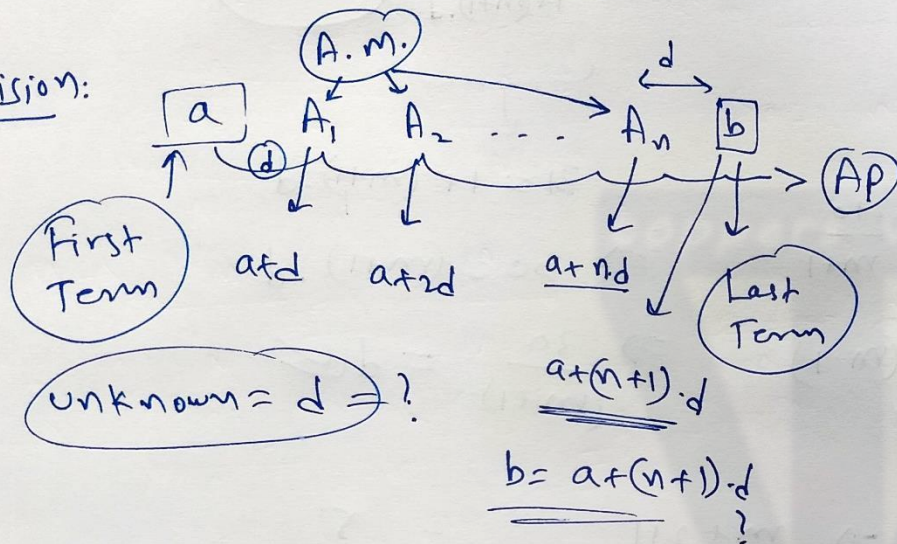
$$\Rightarrow (m-1) \cdot 6 = 156$$

$$\Rightarrow \boxed{m = 27} \quad \checkmark$$

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Revision:



Q.14

Sequence: $8, A_1, A_2, A_3, A_4, A_5, 26$ with common difference d .

Terms and their values:

- 8
- $8 + d = 11$
- $8 + 2d = 14$
- $8 + 3d = 17$
- $8 + 4d = 20$
- $8 + 5d = 23$
- $8 + 6d = 26$

Equation: $26 = 8 + 6d$

Solving for d :

$$\Rightarrow 18 = 6d$$

$$\Rightarrow \boxed{3 = d}$$

Handwritten notes: $C.D = d$ let, 5 and 9 circled.

Q.15

$$a \neq b$$

A.M. between a & $b = \frac{a+b}{2}$

Self

$$\Rightarrow \frac{a+b}{2} = \frac{a+b}{2}$$

$$\Rightarrow \cancel{a^n} + a \cdot b^{n-1} + b \cdot a^{n-1} + \cancel{b^n} = \cancel{a^n} + \cancel{b^n}$$

$$\Rightarrow \cancel{a \cdot b^{n-1}} + \cancel{a \cdot b^{n-1}} = \cancel{a \cdot b^{n-1}} + \cancel{a \cdot b^{n-1}}$$

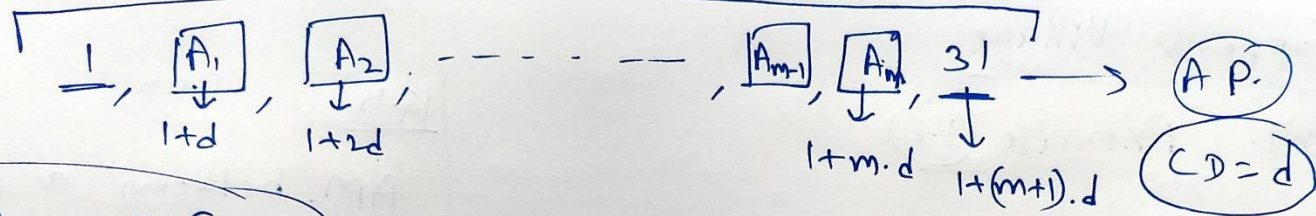
$$\Rightarrow \cancel{a^n} - \cancel{a^{n-1} \cdot b} + \cancel{b^n} - \cancel{a \cdot b^{n-1}} = 0$$

$$\Rightarrow \cancel{a^{n-1}} \cdot \{a - b\} - \cancel{b^{n-1}} \cdot \{a - b\} = 0$$

$$\Rightarrow (a-b) \cdot \{a^{n-1} - b^{n-1}\} = 0$$

Handwritten notes: $a \neq b$, $a^{n-1} = b^{n-1}$, $n-1=0$, $n=1$

Q.16



$$A_1 : A_{m-1} = 5 : 9 \rightarrow m = ?$$

$$A_1 = 1 + 1(d) = 1 + 1 \times \left(\frac{30}{m+1} \right) = 1 + \frac{210}{m+1}$$

$$A_{m-1} = 1 + (m-1)(d) = 1 + \left(\frac{30}{m+1} \right) (m-1)$$

$$31 = 1 + (m+1).d$$

$$\Rightarrow 30 = (m+1).d$$

$$\Rightarrow \frac{30}{(m+1)} = d \checkmark$$

$$\frac{A_1}{A_{m-1}} = \frac{5}{9}$$

$$\Rightarrow \frac{1 + \frac{210}{m+1}}{1 + \frac{30(m-1)}{m+1}} = \frac{5}{9}$$

$$\Rightarrow \frac{\frac{m+1+210}{m+1}}{\frac{m+1+30(m-1)}{m+1}} = \frac{5}{9}$$

$$\Rightarrow \frac{m+211}{31m-29} = \frac{5}{9}$$

$$\Rightarrow 9m + 1899 = 155m - 145$$

$$\Rightarrow 2044 = 146 \text{ (m)}$$

$$\Rightarrow m = \frac{2044}{146} = 14$$

$$\begin{array}{r} 1900 \\ 144 \\ \hline 2044 \end{array}$$

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Q.17

1st instalment $\rightarrow 100$
 2nd instalment $\rightarrow 105$ $\swarrow +5$
 ...
 30th instalment $\rightarrow a_{30}$

$$\begin{aligned} a &= 100 \\ d &= 5 \end{aligned}$$

$$a_{30} = a + 29.d$$

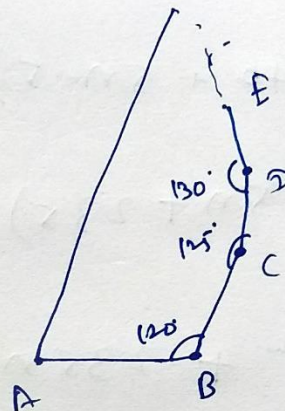
$$a_{30} = 100 + 29 \times 5$$

$$a_{30} = 100 + 145$$

$$\underline{a_{30} = 245}$$

Q.18

Polygon



Angle: $120^\circ, 125^\circ, 130^\circ, 135^\circ, \dots, a_n$

n-sided polygon $\rightarrow$ angles = a_n

★
 Sum of interior angles of
 n-sided polygon $= (n-2) \cdot 180^\circ$

Sum of interior
 Angles $= (n-2) \cdot 180^\circ$

$$\Rightarrow \frac{n}{2} [2 \times 120 + (n-1) \cdot 5] = (n-2) \cdot 180^\circ$$

$$\Rightarrow n(240 + 5n - 5) = 360n - 720^\circ$$

$$\Rightarrow n(240 + 5n - 5) = 360n - 720$$

$$\Rightarrow n(5n + 235) = 360n - 720$$

$$\Rightarrow 5n^2 + 235n = 360n - 720$$

$$\Rightarrow 5n^2 + 235n - 360n + 720 = 0$$

$$\Rightarrow \frac{5n^2 - 125n + 720}{5} = 0$$

$$\Rightarrow \frac{n^2 - 25n + 144}{1} = 0$$

$$\Rightarrow n^2 - 9n - 16n + 144 = 0$$

$$\Rightarrow \cancel{n(n-9)} - (16)(n-9) = 0$$

$$(n-9)(n-16) = 0$$

$$\checkmark \quad \boxed{n=9} \quad \boxed{n=16} \quad \times$$

$$n=9$$

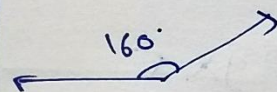
Last Angle = a_9

$$= a + 8d$$

$$= 120 + 8 \times 5$$

$$= 120 + 40$$

$$= 160^\circ$$



$$n=16$$

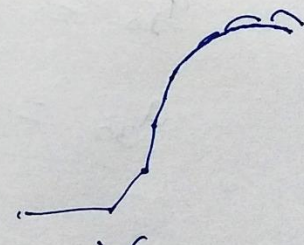
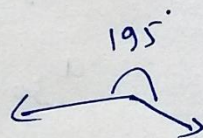
Last angle = a_{16}

$$= a + 15d$$

$$= 120 + 15 \times 5$$

$$= 120 + 75$$

$$= 195^\circ$$



not possible

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Class-11- Maths

Geometric Progression

(G.P.)

ratio of two consecutive terms = Constant (r)

$$\frac{a_{n+1}}{a_n} = r \Rightarrow \boxed{a_{n+1} = r \cdot a_n}, r \neq 0$$

General Term:

(n^{th} term = a_n)

First term = a

Common ratio = $r = \frac{a_2}{a_1} = \frac{a_3}{a_2}$

$$\begin{array}{ccccccc} a & ar & ar^2 & ar^3 & \dots & (a \cdot r^{n-1}) \\ \downarrow & \downarrow & \downarrow & \downarrow & & \uparrow \\ a_1 & a_2 & a_3 & a_4 & & a_n \end{array}$$

General Term

$$\boxed{a_n = a \cdot r^{n-1}} \leftarrow \text{For G.P.}$$

$$\boxed{a_n = a + (n-1) \cdot d} \leftarrow \text{for A.P.}$$

$$\text{Sum of } n\text{-terms of G.P.} = \boxed{S_n = \frac{a(r^n - 1)}{(r - 1)} = \frac{a(1 - r^n)}{(1 - r)}}$$

$$S_n = a + ar + ar^2 + ar^3 + \dots + a \cdot r^{n-1} \quad \text{--- (1)}$$

$$r \cdot S_n = \quad \begin{array}{c} \nearrow \quad \nearrow \quad \nearrow \quad \nearrow \\ ar + ar^2 + ar^3 + \dots + a \cdot r^{n-1} + a \cdot r^n \end{array}$$

$$\underline{\underline{S_n(1 - r) = a - a \cdot r^n = a(1 - r^n)}} \Rightarrow \underline{\underline{S_n(1 - r) = a(1 - r^n)}}$$

Properties of G.P.

① $a_n = a \cdot r^{n-1}$

② $S_n = a \cdot \frac{(r^n - 1)}{(r - 1)}$

③ If a, b, c are in G.P. $\Rightarrow$ $\boxed{b^2 = a \cdot c}$ or $\boxed{b = \sqrt{ac}}$

④

G.P.	$\rightarrow +K$	$\leftarrow$ (in each term)
	$\rightarrow -K$	
	$\rightarrow \times K$	
	$\rightarrow \div K$	

$\left. \begin{array}{l} \rightarrow +K \\ \rightarrow -K \end{array} \right\}$ we do not get a G.P. again.

$\left. \begin{array}{l} \rightarrow \times K \\ \rightarrow \div K \end{array} \right\}$ we get a new G.P.

⑤ Assumption, (we have the product of numbers)

3 No. in G.P. $\rightarrow \frac{a}{r}, a, ar$

4 no. in G.P. $\rightarrow \frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

$\frac{a}{r^3}, \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2, ar^3, \dots$

↑ ↑ × ↑ ↑ ↑ ↑

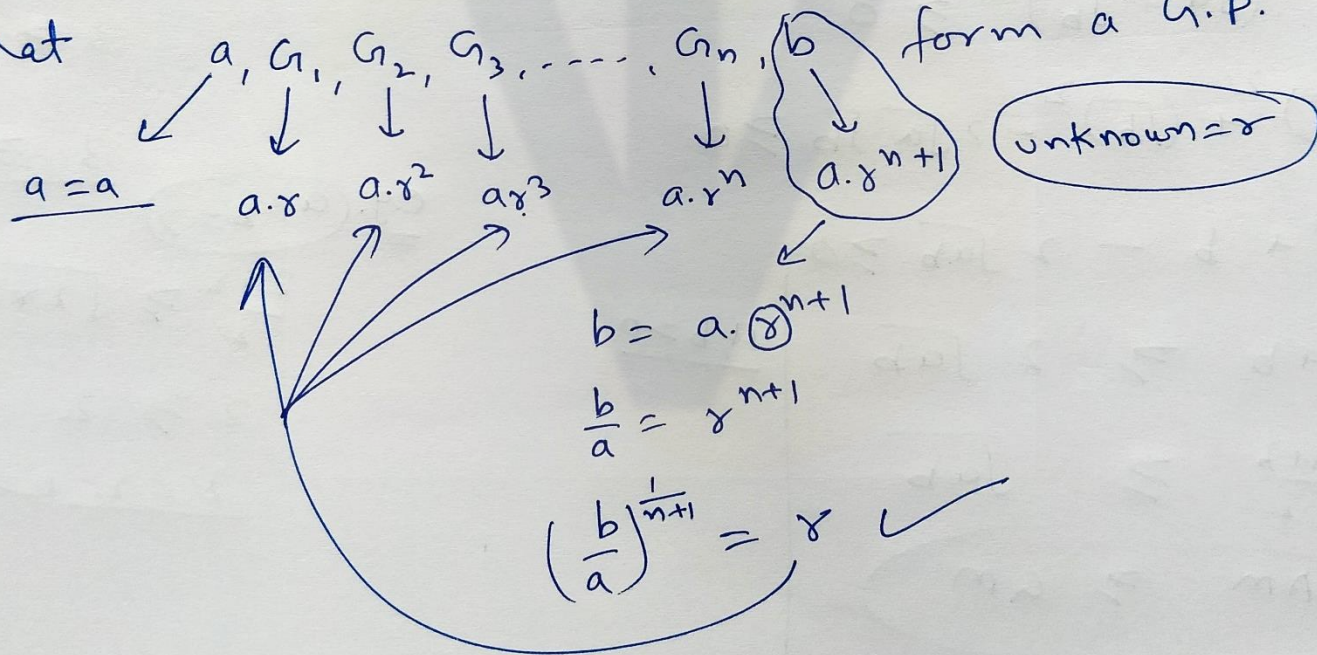
Geometric mean (G.M)

$$\begin{aligned} \text{(i) G.M. of } a, b &= \sqrt{a \cdot b} \\ &= (ab)^{\frac{1}{2}} \end{aligned} \quad \left| \quad \begin{aligned} \text{G.M of } a, b, c &= (a \cdot b \cdot c)^{\frac{1}{3}} = \sqrt[3]{a \cdot b \cdot c} \\ \text{G.M of } \underline{a, b, c, d} &= (a \cdot b \cdot c \cdot d)^{\frac{1}{4}} \end{aligned} \right.$$

(ii) n -G.Ms $\rightarrow$ Find $\rightarrow$ between a & b

Let $\underline{G_1}, \underline{G_2}, \underline{G_3}, \dots, \underline{G_n}$ are the n -Geometric means between a & b

Such that $a, G_1, G_2, G_3, \dots, G_n, b$ form a G.P.



Relation between AM & GM of two positive^{*}
Real numbers (a & b) : $\rightarrow$

$$AM = \frac{a+b}{2}, \quad GM = \sqrt{ab}$$

$$\star \quad AM \geq GM$$

$$\star \quad \frac{a+b}{2} \geq \sqrt{ab}$$

$\hookrightarrow$ e.g. $a=2, b=4$

$$\frac{2+4}{2} \geq \sqrt{2 \times 4}$$

$$\frac{6}{2} \geq \sqrt{8}$$

$$3 \geq 2\sqrt{2}$$

$$2 \times 1.414$$

$$3 \geq 2.828 \dots$$

$$\underline{\text{e.g.}} \quad a=b=2$$

$$\frac{2+2}{2} \geq \sqrt{2 \times 2}$$

$$2 \geq 2$$

$$\underline{2=2} \quad \checkmark$$

Proof.

Square of any Real No. ≥ 0

$$\Rightarrow (\sqrt{a} - \sqrt{b})^2 \geq 0$$

$$\Rightarrow (\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{a}\sqrt{b} \geq 0$$

$$\Rightarrow a + b - 2\sqrt{ab} \geq 0$$

$$\Rightarrow a + b \geq 2\sqrt{ab}$$

$$\underline{a, b > 0}$$

$$\Rightarrow \frac{a+b}{2} \geq \sqrt{ab}$$

$$AM \geq GM$$

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Class - 11 - Maths Exercise (9.3)

Revision:

G.P. ① $a_n = a \cdot r^{n-1}$ ✓

First term = a ② $S_n = \frac{a(r^n - 1)}{(r - 1)}$ ✓
C.R. = r

③ $a, b, c \rightarrow \text{G.P.}$
 $\therefore \boxed{b^2 = a \cdot c}$ ✓

Q.1 G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \dots$

First term = $a = \frac{5}{2}$

C.R. = $r = \frac{a_2}{a_1} = \frac{\cancel{5}/4}{\cancel{5}/2} = \frac{2}{4} = \frac{1}{2}$

20^{th} term = $a_{20} = a \cdot r^{20-1} = \frac{5}{2} \times \left(\frac{1}{2}\right)^{19}$
 $= \frac{5}{2} \times \frac{1}{2^{19}} = \frac{5}{2^{20}}$ ✓

n^{th} term = $a_n = a \cdot r^{n-1}$
 $= \frac{5}{2} \cdot \left(\frac{1}{2}\right)^{n-1} = \frac{5}{2^1 \times 2^{n-1}} = \frac{5}{2^n}$ ✓

Q.2 $a_8 = 192$

✓ $r = 2$

$a_8 = 192$

$\Rightarrow a \cdot r^{8-1} = 192$

$\Rightarrow a \cdot 2^7 = 192$

$\Rightarrow a = \frac{192}{2^7} = \frac{3 \times \cancel{64}}{2 \times \cancel{64}} = \frac{3}{2}$

$a_{12} = a \cdot r^{11}$

$= \frac{3}{2} \times 2^{11}$

$= 3 \times 2^{10} = 3 \times 1024$

$= 3072$ ✓

$a_{12} = ?$

$a_{12} = a \cdot r^{12-1}$
 $= a \cdot r^{11}$

Q.3 $a_5 = p = a.r^4$

$a_8 = q = a.r^7$

$a_{11} = s = a.r^{10}$

Prove $q^2 = ps$

Let first term = a
 $\left. \begin{array}{l} CR = r \end{array} \right\} \Rightarrow (a^m)^n = a^{m.n}$

~~LHS = q^2~~ $q^2 = p.s$

$\Rightarrow (a.r^7)^2 = (a.r^4) \cdot (a.r^{10})$

$\Rightarrow a^2 \cdot (r^7)^2 = a \cdot a \cdot r^4 \cdot r^{10}$

$\Rightarrow \boxed{a^2 \cdot r^{14} = a^2 \cdot r^{14}}$

LHS = RHS ✓

Q.4

$a_4 = (a_2)^2$

First term = $\boxed{-3 = a}$

$a_4 = (a_2)^2$

$\Rightarrow a.r^3 = (a.r)^2$

$\Rightarrow \cancel{a} \cdot \cancel{r}^3 = \cancel{a}^2 \cdot \cancel{r}^2$

$\Rightarrow$

$r = a$

$\Rightarrow \boxed{r = -3}$

$a_7 = ?$

$a.r^6$

$a_7 = a.r^6$

$= (-3)^1 \cdot (-3)^6$

$= (-3)^7$

$= -3^7$

$= -2187$ ✓

Rough,

$\begin{array}{r} 4 \\ 3 \overline{) 81} \\ 3 \overline{) 27} \end{array}$

$3^7 = 3^5 \times 3^2$

$= 243 \times 9$

$= 2187$

Q.5

(a) $2, 2\sqrt{2}, 4, \dots, 128$

$a = 2$

$r = \frac{a_2}{a_1} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$a_n = 128$

$\Rightarrow a \cdot r^{n-1} = 128$

$\Rightarrow 2 \cdot (\sqrt{2})^{n-1} = 2^7$

$\Rightarrow 2^1 \cdot (2^{1/2})^{n-1} = 2^7$

$\Rightarrow 2^1 \cdot 2^{\frac{n-1}{2}} = 2^7$

$\Rightarrow 2^{1 + \frac{n-1}{2}} = 2^7$

$1 + \frac{n-1}{2} = 7$

$\Rightarrow \frac{n-1}{2} = 6$

$\Rightarrow n-1 = 12$

let $a_n = 128$

13th term

$n = 13$

(b) $\sqrt{3}, 3, 3\sqrt{3}, \dots, 729$

$a = \sqrt{3}$

$r = \frac{3}{\sqrt{3}} = \frac{\sqrt{3} \cdot \sqrt{3}}{\sqrt{3}} = \sqrt{3}$

let $a_n = 729$

$\Rightarrow a \cdot r^{n-1} = 729$

$\Rightarrow (\sqrt{3})^1 \cdot (\sqrt{3})^{n-1} = 729$

$\Rightarrow (\sqrt{3})^{1+n-1} = 3^6$

$\Rightarrow (\sqrt{3})^n = 3^6$

$\Rightarrow (3)^{\frac{n}{2}} = 3^6$

$\frac{n}{2} = 6$

$n = 12$

a_n

12th term

Q.5 (c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots, \frac{1}{19683}$

$$a = \frac{1}{3}$$

$$r = \left(\frac{\frac{1}{9}}{\frac{1}{3}} \right) = \frac{3}{9} = \frac{1}{3}$$

Let $a_n = \frac{1}{19683}$

$$\Rightarrow a \cdot r^{n-1} = \frac{1}{19683}$$

$$\Rightarrow \left(\frac{1}{3} \right) \cdot \left(\frac{1}{3} \right)^{n-1} = \frac{1}{19683}$$

$$\Rightarrow \frac{1}{3^1 \times 3^{n-1}} = \frac{1}{19683}$$

$$\Rightarrow \frac{1}{3^{\textcircled{3}}} = \frac{1}{3^{\textcircled{9}}}$$

$n = 9$

↓
9th term

$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$
$3^6 = 729$
$3^7 = 2187$
$3^8 =$
$3^9 = 19683$

↘ x9

Q.6 $-\frac{2}{7}, x, -\frac{7}{2} \rightarrow \underline{\underline{G.P.}}$

$a, b, c \rightarrow aP$

$b^2 = a \times c$

$$x^2 = -\frac{2}{7}x - \frac{7}{2}$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm \sqrt{1}$$

$x = \pm 1$

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Class-11-Maths. Exercise (9.3)

G.P. $a_n = a \cdot r^{n-1}$ $(CR=r)$
 $S_n = \frac{a(r^n - 1)}{(r - 1)} = \frac{a(1 - r^n)}{(1 - r)}$ $(a_1 = a)$

Q.7 0.15, 0.015, 0.0015 20 terms

$$a = 0.15$$

$$r = \frac{0.015}{0.15} = 0.1 = \frac{15}{150} \times \frac{100}{1000}$$

$$n = 20$$

$$S_{20} = \frac{a(1 - r^{20})}{1 - r} = \frac{0.15(1 - 0.1^{20})}{0.9}$$

$$= \frac{0.15 \cdot [1 - (0.1)^{20}]}{1 - 0.1}$$

$$= \frac{0.05}{0.9} \times [1 - (0.1)^{20}]$$

Q.8 $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \dots$ n terms

$$a = \sqrt{7}$$

$$r = \frac{\sqrt{21}}{\sqrt{7}} = \frac{\sqrt{7 \times 3}}{\sqrt{7}} = \sqrt{3}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_n = \frac{\sqrt{7}((\sqrt{3})^n - 1)}{\sqrt{3} - 1}$$

$$\sqrt{3} = 3^{\frac{1}{2}}$$

$$S_n = \frac{\sqrt{7}(3^{\frac{n}{2}} - 1)}{\sqrt{3} - 1}$$

[Q.9] $1, -a, a^2, -a^3, \dots n \text{ terms.}$

$$a = 1$$

$$r = \frac{-a}{1} = -a$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_n = \frac{1.((-a)^n - 1)}{-a - 1}$$

$$S_n = \frac{1 - (-a)^n}{a + 1}$$

Q.10

$x^3, x^5, x^7, \dots n \text{ terms}$

$$a = x^3$$

$$r = \frac{x^5}{x^3} = x^2$$

$$S_n = \frac{a.(r^n - 1)}{r - 1}$$

$$S_n = \frac{x^3 [(x^2)^n - 1]}{x^2 - 1} = \frac{x^3 (x^{2n} - 1)}{x^2 - 1}$$

Q.11

$$\sum_{k=1}^{11} (2+3^k)$$

||

$$= \underbrace{(2+3^1)}_{k=1} + \underbrace{(2+3^2)}_{k=2} + \underbrace{(2+3^3)}_{k=3} + \dots + \underbrace{(2+3^{11})}_{k=11}$$

$$= \underbrace{(2+2+2+\dots+2)}_{11\text{-terms}} + \underbrace{(3^1+3^2+3^3+\dots+3^{11})}_{11\text{-terms}}$$

G.P. $\rightarrow S_n = \frac{a \cdot (r^n - 1)}{r - 1}$

$$\begin{aligned} a &= 3 \\ r &= 3 \\ n &= 11 \end{aligned}$$

$$= 22 + \frac{3 \cdot (3^{11} - 1)}{3 - 1}$$

$$= 22 + \frac{3}{2} \cdot (3^{11} - 1) \quad \checkmark$$

Q. 12

$$a_1 + a_2 + a_3 = S_3 = \frac{39}{10}$$

Their product = 1

$$r = ?$$

Terms = ?

Let

$$\left. \begin{aligned} a_1 &= \frac{a}{r} = \frac{1}{r} \\ a_2 &= a = 1 \\ a_3 &= ar = r \end{aligned} \right\} \text{Common ratio} = r$$

(up.)

$$a, \boxed{\frac{a}{r}, a, ar}, ar^2, ar^3, \dots$$

Given

~~Let~~ their product = 1

$$\Rightarrow a_1 \times a_2 \times a_3 = 1$$

$$\Rightarrow \frac{a}{r} \cdot a \cdot ar = 1$$

$$\Rightarrow a^3 = 1$$

$$\Rightarrow \boxed{a = 1}$$

$$\text{Sum} = a_1 + a_2 + a_3 = \frac{39}{10}$$

$$\Rightarrow \frac{1}{r} + 1 + r = \frac{39}{10}$$

$$\Rightarrow \frac{1+r+r^2}{r} = \frac{39}{10}$$

$$\Rightarrow 10 + 10r + 10r^2 = 39r$$

$$\Rightarrow 10r^2 - 29r + 10 = 0$$

$$\Rightarrow \underbrace{10r^2 - 25r} - \underbrace{4r + 10} = 0$$

$$\Rightarrow 5r(2r-5) - 2(2r-5) = 0$$

$$\Rightarrow (5r-2) \cdot (2r-5) = 0$$

Numbers

$$a_1 = \frac{1}{r} = \frac{1}{\left(\frac{2}{5}\right)} = \frac{5}{2}$$

$$a_2 = 1 = 1$$

$$a_3 = r = \frac{2}{5}$$

$$r = \left(\frac{2}{5}\right), \frac{5}{2}$$

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Class - 11 - Maths Exercise 9.3

G.P.

$$\frac{\text{G.P.}}{a_n = a \cdot r^{n-1}} \quad \checkmark$$

$$S_n = \frac{a \cdot (r^n - 1)}{(r - 1)} \quad \checkmark$$

Q. 13 $3, 3^2, 3^3, \dots, a_n$ $n=?$

$$\text{Sum} = 120 = S_n$$

$$S_n = \frac{3 \cdot (3^n - 1)}{3 - 1} = \frac{40}{2} \quad \left| \begin{array}{l} a=3 \\ r=3 \end{array} \right.$$

$$\Rightarrow \frac{3^n - 1}{2} = 40$$

$$\Rightarrow 3^n - 1 = 80$$

$$\Rightarrow 3^n = 81 = 3^4$$

$$\Rightarrow \boxed{n=4}$$

Q. 14

$$\underbrace{a, ar, ar^2}_{16}, \underbrace{ar^3, ar^4, ar^5}_{128}, ar^6, \dots$$

$$a + ar + ar^2 = 16 \Rightarrow a(1 + r + r^2) = 16 \quad \text{--- (1)}$$

$$ar^3 + ar^4 + ar^5 = 128 \Rightarrow ar^3(1 + r + r^2) = 128 \quad \text{--- (2)}$$

$$\frac{\text{eqn (1)}}{\text{eqn (2)}} \Rightarrow \frac{a(1+r+r^2)}{ar^3(1+r+r^2)} = \frac{16}{128}$$

$$\Rightarrow \left(\frac{1}{r^3} \right) = \frac{1}{8} = \left(\frac{1}{2^3} \right)$$

$$\Rightarrow \boxed{r=2} \quad \checkmark$$

$$a(1 + 2 + 4) = 16$$

$$\xrightarrow{\text{By eqn (1)}} \boxed{a = \frac{16}{7}} \quad \checkmark$$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{\frac{16}{7} \times (2^n - 1)}{2 - 1} = \frac{16}{7} \times (2^n - 1) \quad \checkmark$$

Q.15 G.P. $a = 729$, $a_7 = 64$, $S_7 = ?$

$$a_7 = 64$$

$$\Rightarrow a \cdot r^6 = 64$$

$$\Rightarrow 729 \cdot r^6 = 64$$

$$\Rightarrow r^6 = \frac{64}{729} = \frac{2^6}{3^6} = \left(\frac{2}{3}\right)^6$$

$$\Rightarrow \boxed{r = \frac{2}{3}}$$

$$S_7 = \frac{a \cdot (r^7 - 1)}{r - 1}$$

$$= 729 \cdot \frac{\left(\left(\frac{2}{3}\right)^7 - 1\right)}{\left(\frac{2}{3} - 1\right)} = 729 \cdot \frac{\left(\left(\frac{2}{3}\right)^7 - 1\right)}{-\frac{1}{3}}$$

$$= 2187 \left[1 - \left(\frac{2}{3}\right)^7 \right]$$

Q.16

$$a, ar, ar^2, ar^3, ar^4, ar^5, \dots$$

$\uparrow$ $\uparrow$
 a_3 a_5

$$a + ar = -4$$

$$a_5 = 4 \cdot (a_3) \Rightarrow ar^4 = 4 \cdot ar^2$$

$$r^2 = 4$$

$$\Rightarrow \boxed{r = \pm 2}$$

$$a + ar = -4$$

$$\Rightarrow a(1+r) = -4$$

$$\Rightarrow a = \frac{-4}{1+r}$$

$$\boxed{r = 2}$$

$$a = \frac{-4}{1+2} = -\frac{4}{3}$$

G.P.

$$-\frac{4}{3}, -\frac{8}{3}, -\frac{16}{3}, \dots$$

$$\boxed{r = -2}$$

$$a = \frac{-4}{1+(-2)} = \frac{-4}{-1} = 4$$

$$\underline{\underline{4, -8, 16, \dots}}$$

Q.17

G.P.

$$a_4, a_{10}, a_{16}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\frac{a \cdot r^3}{x}, \frac{a \cdot r^9}{y}, \frac{a \cdot r^{15}}{z}$$

To prove: x, y, z are also in G.P.

$$y^2 = x \cdot z$$

$$\text{LHS} = y^2 = (a \cdot r^9)^2 = a^2 \cdot (r^9)^2 = a^2 \cdot r^{18}$$

$$\text{RHS} = x \cdot z = (a r^3) \cdot (a r^{15}) = a^2 \cdot r^{18}$$

$$\therefore \boxed{y^2 = xz}$$

$\therefore x, y, z$ are in G.P.

Q.18

Sum of n -terms of
8, 88, 888, 8888, ...

$$S_n = 8 + 88 + 888 + \dots +$$

$$= 8(1 + 11 + 111 + \dots)$$

$$= \frac{8}{9}(\underline{9} + \underline{99} + \underline{999} + \dots)$$

$$= \frac{8}{9}[\underline{(10-1)} + \underline{(100-1)} + \underline{(1000-1)} + \dots]$$

$$= \frac{8}{9}[\overbrace{(10 + 100 + 1000 + \dots)}^{\text{G.P.}} - (1 + 1 + 1 + \dots)]$$

$$= \frac{8}{9}[\underbrace{[10^1 + 10^2 + 10^3 + \dots + 10^n]}_{\text{G.P.}} - \underbrace{(1 + 1 + \dots + 1)}_{n \text{ times}}]$$

$$= \frac{8}{9}\left[\frac{10 \cdot (10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{8}{9}\left[\frac{10}{9} \cdot (10^n - 1) - n\right]$$

$$= \frac{80}{81} \cdot (10^n - 1) - \frac{8n}{9} \quad \checkmark$$

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Class - 11 - Maths. Exercise (9.3)

G.P. $a_n = a \cdot r^{n-1}$

$$S_n = \frac{a \cdot (r^n - 1)}{(r - 1)}$$

Q. 19

G.P._I = 2, 4, 8, 16, 32

G.P._{II} = 128, 32, 8, 2, $\frac{1}{2}$

ATQ. G.P. $\rightarrow 256, 128, 64, 32, 16$
 $\uparrow$
 $(a_1), (r = \frac{1}{2})$

Sum = S_5

$$= 256 \cdot \frac{\left(\left(\frac{1}{2}\right)^5 - 1\right)}{\left(\frac{1}{2} - 1\right)} = 256 \cdot \frac{\left(\frac{1}{32} - 1\right)}{-\frac{1}{2}}$$

$$= 256 \cdot \frac{\left(\frac{+31}{32}\right)}{\left(\frac{+1}{2}\right)} \cdot 16 = \frac{16}{256 \times 31}{16} = 496 \checkmark$$

Q. 20

G.P._I $\rightarrow a, ar, ar^2, \dots, ar^{n-1}$

G.P._{II} $\rightarrow A, AR, AR^2, \dots, AR^{n-1}$

Sequence $aA, aArR, aA(rR)^2, aA(rR)^3$
 $\dots; aA(rR)^{n-1}$

G.P. $\leftarrow$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \dots = \text{Common Ratio}$$

$$\frac{aArR}{aA} = \frac{aA(rR)^2}{aA(rR)} = \dots = CR$$

$$\Rightarrow rR = rR = \dots = CR$$

Common Ratio = rR

②1 $a_1, a_2, a_3, a_4 \rightarrow \text{G.P.}$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $a, ar, ar^2, ar^3 \leftarrow \text{Let.}$

$$a_3 = a_1 + 9 \Rightarrow ar^2 = a + 9 \Rightarrow ar^2 - a = 9 \Rightarrow a(r^2 - 1) = 9 \quad \text{--- (1)}$$

$$a_2 = a_4 + 18 \Rightarrow ar = ar^3 + 18 \Rightarrow ar - ar^3 = 18 \Rightarrow -ar(r^2 - 1) = 18 \quad \text{--- (2)}$$

$$\frac{ea^n \text{ (1)}}{ea^n \text{ (2)}} \Rightarrow \frac{\cancel{a}(r^2-1)}{-\cancel{ar}(r^2-1)} = \frac{9}{18}$$

$$\Rightarrow -\frac{1}{r} = \frac{1}{2} \Rightarrow \boxed{r = -2} \rightarrow \text{By } ea^n \text{ (1)}$$

$$a(-2)^2 - 1 = 9$$

$$\Rightarrow a(\cancel{r}) = \cancel{9}_3$$

$$\Rightarrow \boxed{a = 3}$$

Four No. $\boxed{a = 3, r = -2}$

$$a_1 = a = 3$$

$$a_2 = ar = -6$$

$$a_3 = ar^2 = 12$$

$$a_4 = ar^3 = -24$$

Q. 22.

p^{th} term $\rightarrow a = A \cdot R^{p-1}$

$$q^{+n} \text{ term} \rightarrow b = A \cdot R^{2-1}$$

$$r^{\text{th}} \text{ term} \rightarrow C = A \cdot R^{r-1}$$

To Prove $\rightarrow a^{2-r} \cdot b^{r-p} \cdot c^{p-q} = 1$ ✓

$$LHS = a^{q-r} \cdot b^{r-p} \cdot c^{p-q}$$

$$= (A \cdot R)^{p-1} \cdot (A \cdot R)^{q-1} \cdot (A \cdot R)^{r-1}$$

$$= \frac{(q-x) + (x-p) + (p-q)}{A} \times R \frac{pq - px - q + x + qx - pq - x + p + px - qx - p + q}{R}$$

$$= A^0 \times R^0$$

$$= 1 \times 1 = 1 = \text{RHS.}$$



Q. 23 $a_1 = a$
 $a_n = b$

Prove $\rightarrow$
 $P^2 = (ab)^n$

$P =$ product of n terms.

Let Common Ratio $= r$

$a_1 = a$

$a_n = b = a \cdot r^{n-1}$

$P = a_1 \cdot a_2 \cdot a_3 \cdot a_4 \dots a_n$

$P = (a) \cdot (ar) \cdot (ar^2) \cdot (ar^3) \dots (a \cdot r^{n-1})$

$= \underbrace{(a \cdot a \cdot a \dots a)}_{n\text{-times}} \cdot \underbrace{(r^1 \cdot r^2 \cdot r^3 \dots r^{n-1})}_{\text{A.P.}}$

$= (a^n) \cdot \left(r^{1+2+3+\dots+(n-1)} \right)$

$S_{n-1} = \frac{n-1}{2} (2 \times 1 + (n-2) \cdot 1)$

$= \frac{n-1}{2} (2 + n - 2)$

$= \frac{(n-1) \cdot n}{2}$

$P = a^n \cdot r^{\frac{(n-1) \cdot n}{2}}$

LHS $= P^2 = \left[a^n \cdot r^{\frac{n(n-1)}{2}} \right]^2$

$= a^{2n} \cdot r^{n(n-1)} \quad \text{--- (1)}$

RHS $= (ab)^n$
 $= \left(\underline{a} \cdot \underline{a \cdot r^{n-1}} \right)^n$

$= \left[a^2 \cdot r^{n-1} \right]^n$

$= a^{2n} \cdot r^{n(n-1)} \quad \text{--- (2)}$

LHS $=$ RHS

$P^2 = (ab)^n$

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Class-11-Maths. Exercise (9.3)

Q.24

$$\underbrace{\boxed{a}, \boxed{ar}, \boxed{ar^2}, \dots, \boxed{a \cdot r^{n-1}}}_{\text{Sum}_1} \quad \underbrace{\boxed{a \cdot r^n}, \boxed{a \cdot r^{n+1}}, \dots, \boxed{a \cdot r^{2n-1}}}_{\text{Sum}_2}$$

To Prove

$$\frac{\text{Sum}_1}{\text{Sum}_2} = \frac{1}{r^n}$$

$$\text{Sum}_1 = S_n^I = \frac{a \cdot (r^n - 1)}{r - 1}$$

$$\text{Sum}_2 = S_n^{II} = \frac{a \cdot r^n \cdot (r^n - 1)}{r - 1}$$

$$\frac{\text{Sum}_1}{\text{Sum}_2} = \frac{\cancel{a} \cdot \cancel{(r^n - 1)}}{\cancel{a \cdot r^n} \cdot \cancel{(r^n - 1)}} = \frac{1}{r^n}$$

Q.25

$$\begin{matrix} a & b & c & d & \longrightarrow \text{G.P.} \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \text{let } a & ar & ar^2 & ar^3 \end{matrix} \quad \text{Put}$$

To Prove:

$$(a^2 + b^2 + c^2) \cdot (b^2 + c^2 + d^2) = (ab + bc + cd)^2$$

$$\begin{aligned} \text{LHS} &= (a^2 + b^2 + c^2) \cdot (b^2 + c^2 + d^2) \\ &= (a^2 + a^2 r^2 + a^2 r^4) \cdot (a^2 r^2 + a^2 r^4 + a^2 r^6) \\ &= a^2 (1 + r^2 + r^4) \cdot a^2 r^2 (1 + r^2 + r^4) \\ &= a^4 r^2 (1 + r^2 + r^4)^2 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= (ab + bc + cd)^2 \\ &= [a \cdot ar + ar \cdot ar^2 + ar^2 \cdot ar^3]^2 \\ &= [a^2 r + a^2 r^3 + a^2 r^5]^2 \\ &= [r \cdot a^2 \cdot \{1 + r^2 + r^4\}]^2 \\ &= a^4 r^2 (1 + r^2 + r^4)^2 \end{aligned}$$

LHS = RHS

Q.26

$$\begin{array}{ccccccc} 3 & G_1 & G_2 & 81 & \rightarrow & \underline{\underline{G.P.}} \\ & \downarrow & \downarrow & \downarrow & & \\ & 3 \cdot x & 3x^2 & 3x^3 & & \end{array}$$

Let $G_1 = 3x = ? = 3 \times 3 = 9$
 $G_2 = 3x^2 = ? = 3 \cdot (3)^2 = 3 \times 9 = 27$

So $81 = 3x^3$

$\Rightarrow 27 = x^3$

$\Rightarrow 3^3 = x^3$

$3 = x$

Common Ratio

3, 9, 27, 81
G.P.

Q.27 Geometric mean $= \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$
 Given between a & b

$\Rightarrow \sqrt[n]{ab} = \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$

$\Rightarrow (a \cdot b)^{\frac{1}{2}} = \left(\frac{a^{n+1} + b^{n+1}}{a^n + b^n} \right)$

$\Rightarrow \underline{\underline{a^{\frac{1}{2}} \cdot b^{\frac{1}{2}}}} = \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$

$\Rightarrow \underline{\underline{a^{n+1/2} \cdot b^{1/2}}} + a^{1/2} \cdot b^{n+1/2} = \underline{\underline{a^{n+1} + b^{n+1}}}$

$\Rightarrow \frac{a^{1/2} \cdot b^{n+1/2}}{a^{n+1/2} \cdot b^{1/2}} = \frac{a^{n+1} + b^{n+1}}{a^{n+1/2} \cdot b^{1/2}}$

$\Rightarrow \frac{a^{n+1/2}}{b^{n+1/2}} \left(\frac{a^{1/2}}{b^{1/2}} \right) = a^{n+1/2} \left(\frac{a^{1/2}}{b^{1/2}} \right)$

$\Rightarrow \frac{a^{n+1/2}}{b^{n+1/2}} = a^{n+1/2}$
 $1 = 1$

$n + \frac{1}{2} = 0$

$n = -\frac{1}{2}$

$\frac{a^{n+1/2}}{b^{n+1/2}} = \frac{a^{n+1/2}}{b^{n+1/2}}$
 $\frac{a^{n+1/2}}{b^{n+1/2}} = \frac{a^{n+1/2}}{b^{n+1/2}}$

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Class-11-Maths Exercise (9.3)

Q.28 let the no. be a & b.

$$G.M. = \sqrt{ab}$$

ATQ. Sum = $6 \times \overline{G.M.}$

$$\Rightarrow \frac{a+b}{b} = \frac{6\sqrt{ab}}{b}$$

$$\Rightarrow \frac{a}{b} + 1 = \frac{6\sqrt{a} \cdot \sqrt{b}}{\sqrt{b} \cdot \sqrt{b}}$$

$$\Rightarrow \frac{a}{b} + 1 = 6\sqrt{\frac{a}{b}}$$

Let $\frac{a}{b} = x$

$$\Rightarrow x + 1 = 6\sqrt{x}$$

$$\Rightarrow (x+1)^2 = (6\sqrt{x})^2$$

$$\Rightarrow x^2 + 1 + 2x = 36x$$

To Prove $\rightarrow$

$$x = \frac{a}{b} = a:b = \frac{3+2\sqrt{2}}{3-2\sqrt{2}} = 17+12\sqrt{2}$$

$$\Rightarrow x^2 + 2x - 36x + 1 = 0$$

$$\Rightarrow 1 \cdot x^2 - 34x + 1 = 0$$

$$\Rightarrow x = \frac{34 \pm \sqrt{(-34)^2 - 4 \cdot 1 \cdot 1}}{2}$$

$$x = \frac{34 \pm \sqrt{1156 - 4}}{2}$$

$$x = \frac{34 \pm \sqrt{1152}}{2}$$

$$x = \frac{34 \pm \sqrt{4 \cdot 4 \cdot 8 \cdot 9}}{2}$$

$$\frac{3+2\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{(3+2\sqrt{2})^2}{(3)^2 - (2\sqrt{2})^2} = \frac{9+8+12\sqrt{2}}{9-8} = 17+12\sqrt{2}$$

4	1152
4	288
8	72
9	9
	1

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{34 \pm 2 \cdot 2 \cdot 2\sqrt{2} \cdot 3}{2}$$

$$x = 17 \pm 12\sqrt{2} = \frac{a}{b}$$

$$a:b = 17+12\sqrt{2} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$a:b = 17-12\sqrt{2} = \frac{3-2\sqrt{2}}{3+2\sqrt{2}}$$

Q.29 Let 2 positive numbers = x_1, x_2

A.M of x_1 & $x_2 = A$ $x_1 = ?$
 $x_2 = ?$

$$\Rightarrow \frac{x_1 + x_2}{2} = A$$

$$\Rightarrow \boxed{x_1 + x_2 = 2A} \text{--- (1)}$$

G.M of x_1 & $x_2 = G$

$$\Rightarrow \sqrt{x_1 \cdot x_2} = G$$

$$\Rightarrow \boxed{x_1 x_2 = G^2} \text{--- (2)}$$

Quadratic Equation. (Roots = x_1, x_2)

$$x^2 - (\text{Sum of Roots}) \cdot x + (\text{Product of Roots}) = 0$$

$$\Rightarrow x^2 - (x_1 + x_2)x + (x_1 x_2) = 0$$

$$\Rightarrow x^2 - (2A)x + (G^2) = 0$$

Roots = x_1, x_2

$$1. x^2 - 2Ax + G^2 = 0 \quad \begin{matrix} \nearrow x_1 \\ \searrow x_2 \end{matrix}$$

↓ solve (Roots).

Quadratic Form

$$x = \frac{-(-2A) \pm \sqrt{(-2A)^2 - 4 \cdot 1 \cdot G^2}}{2}$$

$$x = \frac{2A \pm \sqrt{4A^2 - 4G^2}}{2}$$

$$x = \cancel{2}A \pm \cancel{2} \sqrt{A^2 - G^2}$$

$$x = A \pm \sqrt{(A+G)(A-G)}$$

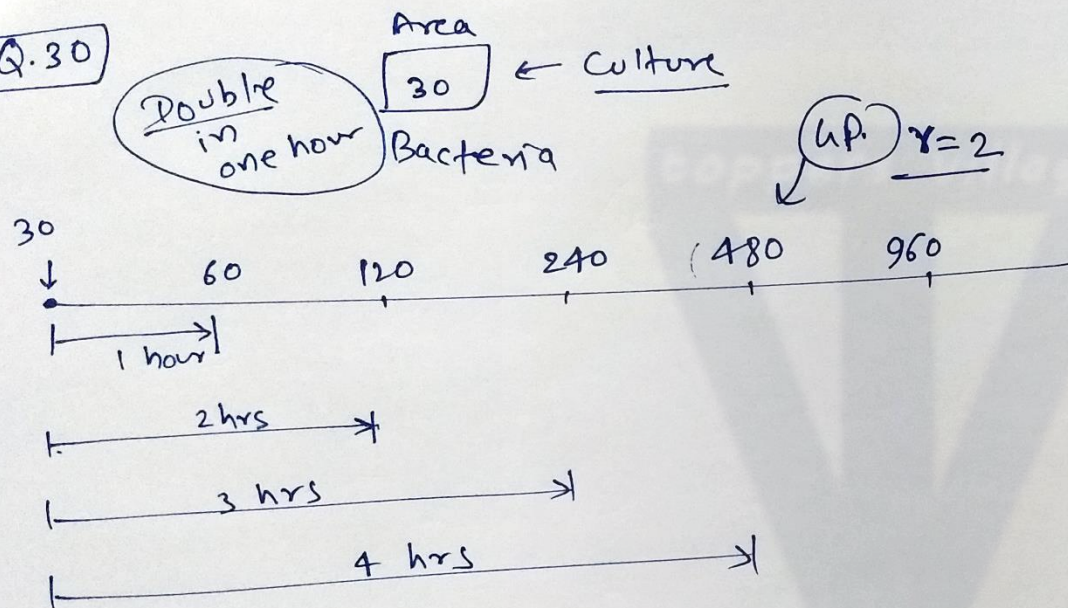
$$x_1 = A + \sqrt{(A+G)(A-G)}$$

$$x_2 = A - \sqrt{(A+G)(A-G)}$$

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Class - 11 - Maths Exercise (9.3)

Q.30



G.P. $a_1, a_2, a_3, a_4, \dots$

$r=2$

$a_1 \downarrow$
 $\textcircled{60}$
 $\uparrow$
 a

$a_2 \downarrow$
 $\textcircled{120}$

$a_3 \downarrow$
 $\textcircled{240}$

$a_4 \downarrow$
 $\textcircled{480}$

$$a_n = a \cdot r^{n-1}$$

$$= a_n = 60 \times (2)^{n-1}$$

At the end of n-hrs.

At the end of

$$2 \text{ hrs} = a_2 = 60 \times (2)^{2-1}$$

$$= 60 \times 2^1$$

$$= 120$$

At the end of

$$4 \text{ hrs} = 60 \times (2)^{4-1}$$

$$= 60 \times 2^3$$

$$= 60 \times 8 = 480$$

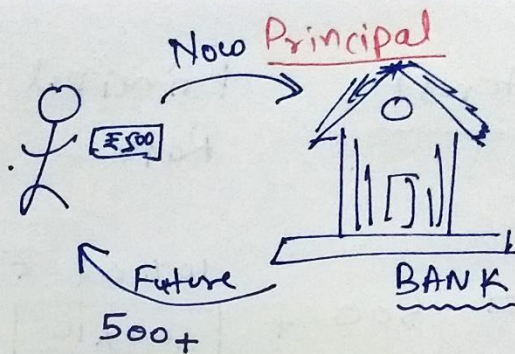
At the end of n hrs.

$$a_n = 60 \times (2)^{n-1}$$

$$= 30 \times 2^1 \times 2^{n-1}$$

$$= 30 \times 2^n \checkmark$$

Q. 31 Principal = ₹ 500
 Rate = 10 %
 Time = 10 years



Simple Interest.

$$P = ₹ 500$$

$$R = 10\%$$

$$\boxed{\text{Amount}} = \text{Principal} + \text{S.I.}$$

or

$$\text{Principal} + \text{C.I.}$$

₹ 500 की 10%.

After one year = $500 + \boxed{500 \times \frac{10}{100}} = 500 + \boxed{50} = 550$

After ~~one~~ 2 years = $500 + \boxed{50} + \boxed{50} = 600$

500 की 10% 500 की 10%

After 3 years = $500 + \boxed{50} + \boxed{50} + \boxed{50} = 650$

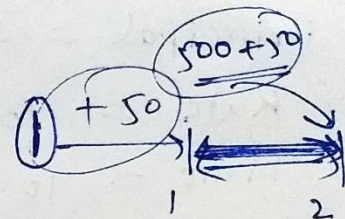
↑
↓
↓

Principal
Simple Interest
Amount.

Compound Interest.

Principal = 500 Rs.

Rate = 10%.



$$\text{After one year} = 500 + \frac{10\% \text{ of } ₹500}{\boxed{500 \times \frac{10}{100}}} = 500 + 50 = \underline{\underline{550}}$$

$$\text{After 2 years} = 500 + \boxed{50} + \frac{10\% \text{ of } ₹550}{\boxed{550 \times \frac{10}{100}}} = 500 + 50 + 55 = \underline{\underline{605}}$$

$$\text{After 3 years} = 500 + \boxed{50} + \boxed{55} + \frac{10\% \text{ of } 605}{\boxed{605 \times \frac{10}{100}}} = 500 + 50 + 55 + 60.5 = ₹665.5$$

Principal = ₹ 500

Rate = 10%.

$$P \left(1 + \frac{R}{100} \right)^n$$

Amount by S.I.

Amount by C.I.

₹ 550.

$a_1 = ₹ 550$

₹ 600

$a_2 = ₹ 605$

₹ 650

$a_3 = ₹ 665.5$

← After one year

← After 2 years

← After 3 years.

AP.

G.P.

Our Question

$$r = \frac{a_2}{a_1} = \frac{605}{550} = \frac{11}{10} = 1.1$$
$$\frac{665.5}{605}$$

Amount

550, 605, 665.5, ...

$r = 1.1$

a_1

a_2

a_3

a_{10}

$$a_{10} = a \cdot r^{10-1}$$

$$= 550 \times (1.1)^9 = 500 \times (1.1)^{10}$$

$$500 \times (1.1) \times (1.1)^9$$

Q.32 Roots of Quadratic eqⁿ. = x_1 & x_2 (Let)

$$\text{Am of Roots} = 8 = \frac{x_1 + x_2}{2} \Rightarrow \boxed{x_1 + x_2 = 16}$$

$$\text{G.M of Roots} = 5 = \sqrt{x_1 \cdot x_2} \Rightarrow \boxed{x_1 \cdot x_2 = 25}$$

Quadratic eqⁿ.

$$x^2 - (\text{Sum of Roots}) \cdot x + \left(\begin{array}{c} \text{product} \\ \text{of} \\ \text{roots} \end{array} \right) = 0$$

$$\Rightarrow x^2 - (x_1 + x_2) \cdot x + x_1 \cdot x_2 = 0$$

$$\Rightarrow \boxed{x^2 - 16x + 25 = 0} \quad \checkmark$$

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Class-11-Maths

Special Series

Ex (9.4)

Sum of n-terms of special Series

① $1+2+3+\dots+n$ = Sum of first n -natural numbers $= \sum_{k=1}^n k = \frac{n(n+1)}{2}$

② $1^2+2^2+3^2+\dots+n^2$ = Sum of squares of first n -natural numbers $= \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$

③ $1^3+2^3+3^3+\dots+n^3$ = Sum of cubes of first n -natural numbers $= \sum_{k=1}^n k^3 = \left[\frac{n(n+1)}{2} \right]^2$

Note: ① Sum of the n -terms of any Series

$$= S_n = a_1 + a_2 + \dots + \underline{a_n}$$

★ $S_n = \sum_{k=1}^n a_k$

② ✓ $\sum_{k=1}^n (a_k \pm t_k) = \sum_{k=1}^n a_k \pm \sum_{k=1}^n t_k$

✓ $\sum_{k=1}^n 3 \cdot a_k = 3 \cdot \sum_{k=1}^n a_k$ Right

$\sum_{k=1}^n a_k \cdot t_k = \sum_{k=1}^n a_k \cdot \sum_{k=1}^n t_k$

wrong.

(e.g.) $(11^3 + 12^3 + 13^3 + \dots + 20^3)$ $\sum_{k=1}^n k^3 = \left[\frac{n(n+1)}{2} \right]^2$ $a_n = \frac{n^2}{2} + \frac{n^3}{2}$

$$= \boxed{(1^3 + 2^3 + \dots + 10^3) + (11^3 + 12^3 + \dots + 20^3)}$$

$$- (1^3 + 2^3 + \dots + 10^3)$$

$$= \left[\frac{10 \cdot (20+1)}{2} \right]^2 - \left[\frac{5 \cdot (10+1)}{2} \right]^2$$

$$= (10 \times 21)^2 - (5 \times 11)^2$$

$$= (210)^2 - (55)^2 = 44100 - 3025$$

$$= 41075 \checkmark$$

(e.g.) Find the sum of n -terms of series $(1) + (1+2) + (1+2+3) + \dots + (\quad)$

$$a_n = (1+2+\dots+n) = \frac{n(n+1)}{2}$$

$$a_n = \frac{n^2+n}{2} = \frac{n^2}{2} + \frac{n}{2} \checkmark$$

$$S_n = \sum_{k=1}^n (a_k)$$

$$S_n = \sum_{k=1}^n \left(\frac{k^2}{2} + \frac{k}{2} \right)$$

$$= \sum_{k=1}^n \frac{k^2}{2} + \sum_{k=1}^n \frac{k}{2}$$

$$= \frac{1}{2} \left(\sum_{k=1}^n k^2 \right) + \frac{1}{2} \left(\sum_{k=1}^n k \right)$$

$$= \frac{1}{2} \left(\frac{n(n+1)(2n+1)}{6} \right) + \frac{1}{2} \left(\frac{n(n+1)}{2} \right)$$

$$= \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4}$$

$$= \underline{\hspace{2cm}}$$

★ Example: [19] Find the sum of n -terms of

the series: $5 + 11 + 19 + 29 + 41 \dots$ $S_n = ?$

Difference $\rightarrow$ $\begin{matrix} \diagdown & \diagdown & \diagdown & \diagdown \\ 6 & 8 & 10 & 12 \end{matrix}$ $\leftarrow$ AP/WP

Diff. 2 $\rightarrow$ $\begin{matrix} \diagdown & \diagdown & \diagdown \\ 2 & 2 & 2 \end{matrix}$

Today's Concept

$$S_n = 5 + 11 + 19 + 29 + \cancel{41} + \dots + a_{n-1} + a_n$$

$\begin{matrix} \nearrow & \nearrow & \nearrow & \nearrow & \nearrow \\ 5 & 11 & 19 & 29 & \dots & a_{n-2} & a_{n-1} & a_n \end{matrix}$

$$0 = 5 + 6 + 8 + 10 + 12 + \dots + () + () - a_n$$

AP.

First term $= a = 6$

C.D. $= d = 2$

no. of terms $= (n-1)$

$$\text{Sum} = S_{n-1}^{\text{AP}} = \frac{n-1}{2} \{ 2 \times 6 + (n-2) \cdot 2 \}$$

$$= \frac{n-1}{2} \cdot \{ 12 + 2n - 4 \}$$

$$= (n-1) \cdot (n+4) = n^2 + 3n - 4$$

$$\Rightarrow 0 = 5 + (n^2 + 3n - 4) - a_n$$

$$\Rightarrow \boxed{a_n = n^2 + 3n + 1}$$

$$a_n = n^2 + 3n + 1$$

$$S_n = \sum_{k=1}^n \textcircled{a_k} = \sum_{k=1}^n (k^2 + 3k + 1)$$

$$\textcircled{1 = 1 \times 1}$$

$$S_n = \sum_{k=1}^n \textcircled{k^2} + 3 \cdot \sum_{k=1}^n k + \sum_{k=1}^n 1$$

$$\sum_{k=1}^n 1 = \underbrace{1 + 1 + 1 + \dots + 1}_{n\text{-times}} = n$$

$$S_n = \frac{\boxed{n(n+1)(2n+1)}}{6} + 3 \cdot \frac{\boxed{n(n+1)}}{2} + \boxed{n}$$

$$S_n = n \left\{ \frac{2n^2 + 3n + 1}{6} + \frac{3n + 3}{2} + \frac{1}{1} \right\}$$

$$S_n = n \cdot \left\{ \frac{2n^2 + 3n + 1 + 9n + 9 + 6}{6} \right\} = n \cdot \left\{ \frac{n^2 + 6n + 8}{\cancel{6}_3} \right\}$$

$$S_n = \frac{n(n^2 + 6n + 8)}{3} = \frac{n(n+2)(n+4)}{3}$$

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Class - 11 - Maths Exercise (9.4)

Q.1

$$\underbrace{1 \times 2}_{a_1} + \underbrace{2 \times 3}_{a_2} + \underbrace{3 \times 4}_{a_3} + \underbrace{4 \times 5}_{a_4} + \dots$$

$$n^{\text{th}} \text{ term} = a_n = n \cdot (n+1)$$

$$a_n = n^2 + n$$

$$\text{Sum of } n\text{-terms} = S_n = \sum_{k=1}^n a_k$$

$$S_n = \sum_{k=1}^n (k^2 + k)$$

$$= \sum_{k=1}^n k^2 + \sum_{k=1}^n k$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2} \cdot \left\{ \frac{2n+1}{3} + 1 \right\} = \frac{n(n+1)(2n+4)}{2 \times 3}$$

Q.2

$$\underbrace{1 \times 2 \times 3}_{a_1} + \underbrace{2 \times 3 \times 4}_{a_2} + \underbrace{3 \times 4 \times 5}_{a_3} + \dots$$

$$n^{\text{th}} \text{ term} = a_n = n \cdot (n+1) \cdot (n+2)$$

$$S_n = \sum_{k=1}^n a_k = \sum_{k=1}^n k(k+1)(k+2)$$

$$S_n = \sum_{k=1}^n k(k^2 + 3k + 2)$$

$$S_n = \sum_{k=1}^n (k^3 + 3k^2 + 2k)$$

$$S_n = \sum_{k=1}^n k^3 + 3 \sum_{k=1}^n k^2 + 2 \sum_{k=1}^n k$$

$$S_n = \left[\frac{n(n+1)}{2} \right]^2 + 3 \cdot \frac{n(n+1)(2n+1)}{6} + 2 \cdot \frac{n(n+1)}{2}$$

Q.3

$$\underbrace{3 \times 1^2}_{a_1} + \underbrace{5 \times 2^2}_{a_2} + \underbrace{7 \times 3^2}_{a_3} + \dots$$

$$n^{\text{th}} \text{ term} = a_n = (2n+1) \times n^2$$

$$\boxed{a_n = 2n^3 + n^2} \quad \checkmark$$

$$S_n = \sum_{k=1}^n a_k = \sum_{k=1}^n (2k^3 + k^2)$$

$$S_n = 2 \left(\sum_{k=1}^n k^3 \right) + \left(\sum_{k=1}^n k^2 \right)$$

$$S_n = 2 \cdot \left(\frac{n(n+1)}{2} \right)^2 + \frac{n(n+1)(2n+1)}{6}$$

$$S_n = \cancel{2} \cdot \frac{n^2 \cdot (n+1)^2}{\cancel{2}} + \frac{n(n+1)(2n+1)}{6}$$

$$S_n = \frac{n(n+1)}{2} \cdot \left\{ \frac{n(n+1)}{1} + \frac{2n+1}{3} \right\}$$

odd no.
 $\boxed{3, 5, 7, \dots}$

$$\begin{array}{c} \boxed{2n-1} \\ \boxed{1} \\ \boxed{3} \\ \boxed{5} \\ \vdots \end{array} \quad \begin{array}{c} \boxed{2n+1} \\ \boxed{3} \\ \boxed{5} \\ \boxed{7} \\ \vdots \end{array}$$

$$S_n = \frac{n(n+1)}{2} \cdot \left\{ \frac{3(n^2+n)+2n+1}{3} \right\}$$

$$= \frac{n(n+1)}{2} \cdot \left\{ \frac{3n^2+3n+2n+1}{3} \right\}$$

$$= \frac{n(n+1)}{2} \cdot \left(\frac{3n^2+5n+1}{3} \right)$$

$$= \frac{n(n+1) \cdot (3n^2+5n+1)}{6}$$

Class-11-Maths Exercise (9.4)

Q.4

★

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots \text{upto } n\text{-terms}$$

$a_1 \quad a_2 \quad a_3$

$$n^{\text{th}} \text{ term} = (a_n) = \frac{1}{n \times (n+1)} = \frac{1}{n(n+1)}$$

$$\text{Sum of } n\text{-terms} = \left[S_n = \sum_{k=1}^n (a_k) \right]$$

$$\Rightarrow S_n = \sum_{k=1}^n \frac{1}{\underset{\substack{\uparrow \\ \text{First}}}{k} \cdot \underset{\substack{\uparrow \\ \text{Last}}}{(k+1)}}$$

Difference

$$\text{of first \& last} = (k+1) - k$$

$$= \textcircled{1} \rightarrow \text{Numerator}$$

$$\Rightarrow S_n = \sum_{k=1}^n \frac{(k+1) - k}{(k \cdot (k+1))}$$

$$S_n = \sum_{k=1}^n \frac{(k+1) - k}{(k \cdot (k+1))}$$

$$= \sum_{k=1}^n \left\{ \frac{1}{k} - \frac{1}{k+1} \right\}$$

$$= \left\{ \frac{1}{1} - \frac{1}{2} \right\}$$

$$+ \left\{ \frac{1}{2} - \frac{1}{3} \right\}$$

$$+ \left\{ \frac{1}{3} - \frac{1}{4} \right\}$$

⋮

$$+ \left\{ \frac{1}{n} - \frac{1}{n+1} \right\}$$

$$= 1 - \frac{1}{n+1}$$

$$= \frac{(n+1) - 1}{n+1} = \frac{n}{n+1}$$

Q.5

$$[5^2 + 6^2 + 7^2 + \dots + 20^2]$$

$$= \left[(1^2 + 2^2 + 3^2 + 4^2) + [5^2 + 6^2 + \dots + 20^2] \right] - (1^2 + 2^2 + 3^2 + 4^2)$$

$$= (1^2 + 2^2 + \dots + 20^2) - (1^2 + 2^2 + \dots + 4^2)$$

$$= \frac{20 \cdot (20+1) \cdot (20 \times 2 + 1)}{6} - \frac{4 \cdot (4+1) \cdot (2 \times 4 + 1)}{6}$$

$$= \frac{\overset{10}{20} \times \overset{7}{21} \times 41}{\cancel{6}_3} - \frac{\overset{2}{4} \times 5 \times \cancel{9}_3}{\cancel{6}_3}$$

$$= 2870 - 30$$

$$= 2840 \quad \checkmark$$

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Class-11-Maths. Exercise (9.4)

Q.6

$$\underbrace{3 \times 8}_{a_1} + \underbrace{6 \times 11}_{a_2} + \underbrace{9 \times 14}_{a_3} + \dots + \underbrace{(3n) \cdot (3n+5)}_{a_n}$$

$$\begin{array}{ccccccc} & 3, & 6, & 9, & \dots, & 3n \\ \uparrow & \uparrow & \uparrow & & & \uparrow \\ \textcircled{1} & \textcircled{2} & \textcircled{3} & & & \textcircled{n} \end{array}$$

$$\begin{array}{ccccccc} & 8, & 11, & 14, & \dots, & A_n \\ \uparrow & \uparrow & \uparrow & & & \uparrow \\ & +3 & +3 & & & \textcircled{AP} \end{array}$$

$$A_n = a + (n-1)d$$

$$8 + (n-1) \cdot 3 = 8 + 3n - 3$$

$$= 3n + 5$$

$$n^{\text{th}} \text{ term} = \textcircled{a_n} = 3n \cdot (3n+5) = \textcircled{9n^2 + 15n}$$

$$S_n = \sum_{k=1}^n \textcircled{a_k} = \sum_{k=1}^n (9k^2 + 15k)$$

$$S_n = 9 \left(\sum_{k=1}^n k^2 \right) + 15 \left(\sum_{k=1}^n k \right)$$

$$= \underline{9} \cdot \frac{n(n+1)(2n+1)}{\underline{6}} + \underline{15} \frac{n(n+1)}{\underline{2}}$$

$$S_n = \frac{3 \cdot n(n+1)}{2} \cdot \left\{ \frac{3(2n+1)}{2} + 5 \right\}$$

$$S_n = \frac{3n(n+1)}{2} \cdot \frac{n+3}{2}$$

$$S_n = 3 \cdot n \cdot (n+1) \cdot (n+3) \checkmark$$

Q.7 $\underbrace{(1^2)}_{a_1} + \underbrace{(1^2+2^2)}_{a_2} + \underbrace{(1^2+2^2+3^2)}_{a_3} + \dots + \underbrace{(1^2+2^2+\dots+n^2)}_{a_n}$

$$a_n = (1^2+2^2+3^2+\dots+n^2) = \frac{n(n+1)(2n+1)}{6} = \frac{n(2n^2+3n+1)}{6} = \frac{2n^3}{6} + \frac{3n^2}{6} + \frac{n}{6}$$

$$a_n = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$S_n = \sum_{k=1}^n a_k$$

$$S_n = \sum_{k=1}^n \left(\frac{k^3}{3} + \frac{k^2}{2} + \frac{k}{6} \right)$$

$$S_n = \frac{1}{3} \left(\sum_{k=1}^n k^3 \right) + \frac{1}{2} \left(\sum_{k=1}^n k^2 \right) + \frac{1}{6} \left(\sum_{k=1}^n k \right)$$

$$S_n = \frac{1}{3} \cdot \left(\frac{n(n+1)}{2} \right)^2 + \frac{1}{2} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{1}{6} \cdot \frac{n(n+1)}{2}$$

$$= \frac{n^2 \cdot (n+1)^2}{12} + \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{12}$$

$$S_n = \frac{n(n+1)}{12} \cdot \left\{ \frac{n(n+1)}{2} + \frac{(2n+1)}{2} + 1 \right\}$$

$$S_n = \frac{n(n+1)}{12} \cdot \{ n^2 + n + 2n + 1 + 1 \}$$

$$S_n = \frac{n(n+1)}{12} \cdot \{ n^2 + 3n + 2 \}$$

$$S_n = \frac{n(n+1) \cdot (n+1)(n+2)}{12}$$

$$S_n = \frac{n(n+1)^2 \cdot (n+2)}{12}$$

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Class-11-Maths. Exercise (9.4)

[Q.8] n^{th} - term = $(a_n) = \underline{n(n+1)(n+4)} = n(n^2+5n+4)$

$$S_n = \sum_{k=1}^n (a_k) = \sum_{k=1}^n (k^3 + 5k^2 + 4k) \Rightarrow n^3 + 5n^2 + 4n$$

$$\begin{matrix} 3 \times 2 \times 17 \\ 5 \times 3 \times 4 \end{matrix}$$

$$S_n = \sum_{k=1}^n k^3 + 5 \sum_{k=1}^n k^2 + 4 \sum_{k=1}^n k$$

$$S_n = \left[\frac{n(n+1)}{2} \right]^2 + 5 \cdot \left(\frac{n(n+1)(2n+1)}{6} \right) + 4 \left(\frac{n(n+1)}{2} \right)$$

$$S_n = n(n+1) \left\{ \frac{n(n+1)}{4} + \frac{5(2n+1)}{6} + \frac{2}{1} \right\}$$

$$S_n = n(n+1) \cdot \left\{ \frac{3n^2 + 3n + 20n + 10 + 24}{12} \right\}$$

$$S_n = \frac{n(n+1) \cdot (3n^2 + 23n + 34)}{12}$$

$$S_n = \frac{n(n+1) \cdot (3n^2 + 6n + 17n + 34)}{12}$$

$$S_n = \frac{n(n+1) \cdot [3n(n+2) + 17(n+2)]}{12}$$

$$S_n = \frac{n(n+1)(n+2)(3n+17)}{12}$$



Q.9 nth term = $a_n = n^2 + 2^n$

$$S_n = \sum_{k=1}^n (a_k) = \sum_{k=1}^n (k^2 + 2^k) = \sum_{k=1}^n (k^2) + \sum_{k=1}^n (2^k)$$



$$\frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n (2^k) = (2^1) + (2^2) + \dots + (2^n)$$

$$= [2^1 + 2^2 + 2^3 + \dots + 2^n] \text{ G.P.}$$

$$= \frac{a(r^n - 1)}{r - 1}$$

$$a = 2$$

$$r = 2$$

$$\text{no. of terms} = n$$

$$= \frac{2 \cdot (2^n - 1)}{2 - 1} \rightarrow 1$$

$$= \frac{2 \cdot (2^n - 1)}{1}$$

$$= 2^{n+1} - 2$$

Q.10) $n^{\text{th}} \text{ term} = a_n = (2n-1)^2$

$$\Rightarrow a_n = (2n)^2 + (1)^2 - 2(2n)(1)$$

$$\Rightarrow \boxed{a_n = 4n^2 + 1 - 4n}$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

Sum of n -terms = $S_n = \sum_{k=1}^n a_k = \sum_{k=1}^n (4k^2 + 1 - 4k)$

$$S_n = 4 \sum_{k=1}^n k^2 + \sum_{k=1}^n 1 - 4 \sum_{k=1}^n k$$

$$\sum_{k=1}^n 1 = \underbrace{1+1+\dots+1}_{n\text{-times}} = n$$

$$S_n = \cancel{4} \cdot \frac{n(n+1)(2n+1)}{3} + n - \cancel{4} \cdot \frac{n(n+1)}{2}$$

$$S_n = n \cdot \left\{ \frac{2}{3} (2n^2 + 3n + 1) + \frac{1}{1} - \frac{2(n+1)}{1} \right\}$$

$$S_n = n \cdot \left\{ \frac{4n^2 + \cancel{6n} + 2 + 3 - \cancel{6n} - 6}{3} \right\}$$

$$\Rightarrow S_n = \frac{n \cdot (4n^2 - 1)}{3}$$

$$S_n = \frac{n \cdot [(2n)^2 - 1^2]}{3}$$

$$\boxed{S_n = \frac{n(2n+1)(2n-1)}{3}}$$

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Class - 11 - Maths Miscellaneous Exercise

Chapter (9)

Q.1 AP.

To Prove

$$\boxed{\begin{matrix} (m+n)^{\text{th}} \text{ term} \\ + \\ (m-n)^{\text{th}} \text{ term} \end{matrix} = 2(m^{\text{th}} \text{ term})}$$

Proof: first term = a (let) $a_n = a + (n-1) \cdot d$
C.D. = d *

$$\text{LHS} = \underline{(m+n)^{\text{th}} \text{ term}} + \underline{(m-n)^{\text{th}} \text{ term}}$$

$$= a_{m+n} + a_{m-n}$$

$$= \underline{a + (m+n-1) \cdot d} + \underline{a + (m-n-1) \cdot d}$$

$$= 2a + \underline{(m+n-1+m-n-1)} \cdot d$$

$$= 2a + (2m-2) \cdot d$$

$$= \underline{2a} + \underline{2(m-1) \cdot d} = 2 \{ \underline{a + (m-1) \cdot d} \}$$

$$= 2 \{ a_m \}$$

$$= 2 (m^{\text{th}} \text{ term})$$

$$= \text{RHS.}$$

Q.2 Sum of 3 No. in AP = 24
their Product = 440.

Let the numbers be
 $a-d, a, a+d,$

ATQ. $(a-d) + (a) + (a+d) = 24$

$$\Rightarrow 3a = 24 \quad 8$$

$$\boxed{a=8}$$

No. $\rightarrow 8-d, 8, 8+d$

ATQ. $(8-d) \cdot 8 \cdot (8+d) = \overset{55}{\cancel{440}}$

$$\Rightarrow 8^2 - d^2 = 55$$

$$\Rightarrow 64 - \underline{d^2} = 55$$

$$\Rightarrow \boxed{9 = d^2} \Rightarrow \boxed{d = \pm 3}$$

$$a=8, d=+3 \quad (\text{you can also take } -3=d)$$

No. $a-d=5$
 $a=8$
 $a+d=11$

$$\begin{pmatrix} 11 \\ 8 \\ 5 \end{pmatrix}$$

Q.3 AP. Let $\begin{pmatrix} a_1 = a \\ d = d \end{pmatrix}$

$$\text{Sum of } n \text{ terms} = S_n = S_1 = \frac{n}{2} (2a + (n-1) \cdot d)$$

$$\text{Sum of } 2n \text{ terms} = S_{2n} = S_2 = \frac{2n}{2} (2a + (2n-1) \cdot d)$$

$$\text{Sum of } 3n \text{ terms} = S_{3n} = S_3 = \frac{3n}{2} (2a + (3n-1) \cdot d)$$

To Prove: $S_3 = 3(S_2 - S_1)$

$$\begin{aligned} \text{RHS} &= 3(S_2 - S_1) \\ &= 3 \left\{ \frac{2n}{2} (2a + (2n-1) \cdot d) - \frac{n}{2} (2a + (n-1) \cdot d) \right\} \end{aligned}$$

$$= 3 \frac{n}{2} \cdot \{ \underline{4a} + \underline{4nd} - \underline{2d} - \underline{2a} - \underline{nd} + \underline{d} \}$$

$$= \frac{3n}{2} \cdot (2a + \underline{3nd} - d)$$

$$= \frac{3n}{2} \cdot \{ 2a + (3n-1)d \}$$

$$= S_3$$

$$= \text{LHS.}$$

Q.4

between 200 & 400

Divisible by 7

AP. $n=?$ $a=203$

Sequence: $\rightarrow (203, 210, 217, \dots, 399) \rightarrow d=7$

$$\begin{array}{r} 7 \overline{)200} 28 \\ -14 \\ \hline 60 \\ -56 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 7 \overline{)400} 57 \\ -35 \\ \hline 50 \\ -49 \\ \hline 1 \end{array}$$

$$200 = 7 \times 28 + 4$$

$$400 = 7 \times 57 + 1$$

$$\begin{array}{r} 200 \\ -4 \\ \hline 196 \end{array} + 7 = \underline{\underline{203}}$$

$$\begin{array}{r} 400 \\ -1 \\ \hline 399 \end{array}$$

$$S_n = \frac{n}{2} (2a + (n-1) \cdot d)$$

$$S_{29} = \frac{29}{2} \cdot (2 \times 203 + 28 \cdot 7)$$

$$S_{29} = 29 \cdot (203 + 98) = 29(301) = 8729 \checkmark$$

let $399 = n^{\text{th}}$ term

$$399 = a_n$$

$$\Rightarrow 399 = a + (n-1) \cdot d$$

$$\Rightarrow 399 = 203 + (n-1) \cdot 7$$

$$\Rightarrow 196 = (n-1) \cdot 7$$

$$\Rightarrow 28 = n-1 \Rightarrow \underline{\underline{29 = n}}$$

Q.5 Integers from 1 to 100

Divisible by '2' or '5'

Sum = ?

Main Sequence

2, 4, 5, 6, 8, 10, 12, ...

Actual

$$S_n = \frac{n}{2} (2a + (n-1) \cdot d)$$

Divisibly by '2' $\rightarrow 2, 4, 6, 8, 10, 12, \dots, 96, 98, 100 \rightarrow S_1 = S_1 = \frac{50}{2} (4 + 49 \times 2)$

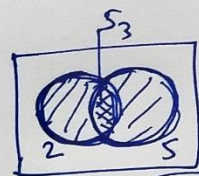
Divisibly by '5' $\rightarrow 5, 10, 15, 20, 25, \dots, 90, 95, 100 \rightarrow S_2 = \frac{20}{2} (10 + 19 \times 5)$

Common No.

Divisibly by

'2' & '5'

= 10



$$S_1 + S_2 - S_3$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Required Answer = $S_1 + S_2 - S_3$

$$= 25(4 + 98) + 10(10 + 95) - 5(20 + 90)$$

$$= 25(102) + 10(105) - 5(110)$$

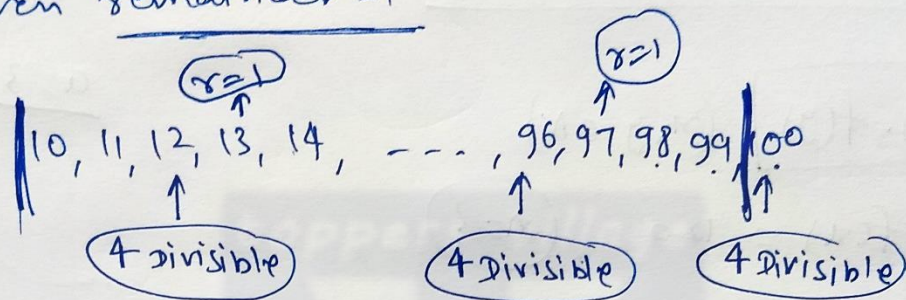
$$= 2550 + 1050 - 550$$

$$= 3050 \checkmark$$

Q.6 Sum of all two Digit Numbers = ?

Divisor = 4 then remainder = 1

10 $\longleftrightarrow$ 99



Required Sequence : $\rightarrow 13, 17, 21, \dots, 97,$

$$a = 13$$

$$d = 4$$

$$n = ? = 22$$

Sum = ?

$$S_n = \frac{n}{2} (2a + (n-1) \cdot d)$$

$$S_n = S_{22} = \frac{22}{2} (26 + 21 \times 4)$$

$$S_{22} = 11 (26 + 84)$$

$$= 11 (110) = 1210 \checkmark$$

let $97 = a_n$

$$\Rightarrow 97 = a + (n-1) \cdot d$$

$$\Rightarrow 97 = 13 + (n-1) \cdot 4$$

$$\Rightarrow 84 = (n-1) \cdot 4$$

$$21 = n-1$$

$$\Rightarrow 22 = n$$

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Class - 11 - Maths Miscellaneous Exercise (9)

Q.7 $f(x+y) = f(x) \cdot f(y) \quad (x, y \in \mathbb{N})$

$$f(1) = 3$$

$$\sum_{x=1}^n f(x) = 120, \quad (n=?)$$

$$\Rightarrow \underbrace{f(1)}_3 + \underbrace{f(2)}_{3^2} + \underbrace{f(3)}_{3^3} + \dots + \underbrace{f(n)}_{3^n} = 120$$

$n=1 \Rightarrow$
 $f(2) = f(1+1) = f(1) \cdot f(1) = 3 \times 3 = 9 = 3^2$

$$f(3) = f(1+2) = f(1) \cdot f(2) = 3 \times 9 = 3^3$$

$$f(4) = 3^4$$

Similarly, $f(n) = 3^n$

$$a=3, r=3,$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow \overbrace{3 + 3^2 + 3^3 + \dots + 3^n}^{\text{Sum}} = 120$$

(G.P.)

$$\Rightarrow \frac{3(3^n - 1)}{3 - 1} = 120$$

$$\Rightarrow 3^n - 1 = 80$$

$$\Rightarrow 3^n = 81 = 3^4$$

$$\Rightarrow n=4$$

Q.8

Sum of Some terms of G.P. = 315

$a = 5$, $r = 2$, $\downarrow$ let

last term

$a_n = ?$

No. of terms

$(n) = ? = 6$

A.T.Q

$$S_n = 315$$

$$\Rightarrow \frac{a(r^n - 1)}{r - 1} = 315$$

$$\Rightarrow \frac{5(2^n - 1)}{2 - 1} = \frac{63}{1}$$

$$\Rightarrow 2^n - 1 = 63$$

$$\Rightarrow 2^n = 64 = 2^6$$

$\boxed{n = 6}$

$$\begin{aligned} a_n &= a \cdot r^{n-1} \\ &= 5 \times (2)^{6-1} \\ &= 5 \times 2^5 \\ &= 5 \times 32 \\ &= 160 \end{aligned}$$

Q.9

$a = 1$

$r = ?$

$$a_3 + a_5 = 90$$

$$\Rightarrow a \cdot r^2 + a \cdot r^4 = 90$$

$$\Rightarrow 1 \cdot r^2 + 1 \cdot r^4 = 90$$

$$\Rightarrow r^2 + r^4 - 90 = 0$$

$$\Rightarrow (r^2)^2 + (r^2) - 90 = 0$$

$r^2 = x$ let

$$\text{then } x^2 + x - 90 = 0$$

$$\Rightarrow x^2 + 10x - 9x - 90 = 0$$

$$\Rightarrow x(x + 10) - 9(x + 10) = 0$$

$$\Rightarrow (x + 10)(x - 9) = 0$$

$$\downarrow$$

~~$x = -10$~~
 $x = -10$

$$\downarrow$$

$x = 9$

$$r^2 = 9$$

$$\boxed{r = \pm 3}$$

$$\boxed{r = \pm 3}$$

~~reject~~

$$\boxed{r^2 = -10}$$

$\oplus \times \ominus$

Q.10 Sum of 3 No. in G.P. = $56 = a + ar + ar^2$ ~~$\Rightarrow$~~ $\Rightarrow 56 = a(1+r+r^2)$ ①

$(-1), (-7), (-21)$

$a = ?$
 $r = ?$

Let $a, ar, ar^2 \leftarrow$ A.P.

$a-1, ar-7, ar^2-21 \rightarrow$ A.P.

Note: If a, b, c are in A.P.
then $2b = a + c$

$\therefore 2(ar-7) = a-1 + (ar^2-21)$

$\Rightarrow 2ar - 14 = a - 1 + ar^2 - 21$

$\Rightarrow 22 - 14 = a + ar^2 - 2ar$

$\Rightarrow 8 = a(1+r^2-2r)$ ②

By $\frac{eq^n ①}{eq^n ②} \Rightarrow$

$\Rightarrow \frac{56}{8} = \frac{a(1+r+r^2)}{a(1+r^2-2r)}$

$\Rightarrow 7 = \frac{1+r+r^2}{1+r^2-2r}$

$\Rightarrow 7 + 7r^2 - 14r = 1 + r + r^2$

$\Rightarrow (6r^2 - 15r + 6 = 0) / 3$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow 2r^2 - 4r - r + 2 = 0$

$\Rightarrow 2r(r-2) - (r-2) = 0$

$\Rightarrow (2r-1)(r-2) = 0$

$r = \frac{1}{2}, r = 2$

$$\text{eqn. (I)}$$

$$r = \frac{1}{2}$$

$$S_6 = a \left(1 + \frac{1}{2} + \frac{1}{4} \right)$$

$$S_6 = a \left(\frac{4+2+1}{4} \right)$$

$$8S_6 = a \cdot \frac{7}{4}$$

$$\boxed{a = 32}$$

Numbers

$$a = 32$$

$$ar = 32 \times \frac{1}{2} = 16$$

$$ar^2 = 32 \times \frac{1}{4} = 8$$

$$\boxed{32, 16, 8}$$

$$S_6 = a(1+r+r^2)$$

$$r = 2$$

$$S_6 = a(1+2+4)$$

$$\boxed{a = 8}$$

Numbers

$$a = 8$$

$$ar = 8 \times 2 = 16$$

$$ar^2 = 8 \times 4 = 32$$

$$\boxed{8, 16, 32}$$

$$\boxed{\text{Q.11}} \quad \text{Total even no. of terms} = \underline{2n}$$

(G.P.)

$$\begin{array}{ccccccccc} a_1 & a_2 & a_3 & a_4 & a_5 & \dots & a_{2n-1} & a_{2n} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow \\ a & ar & ar^2 & ar^3 & ar^4 & \dots & ar^{2n-2} & ar^{2n-1} \end{array}$$

(A.T.Q.)

$$\underbrace{(a_1 + a_2 + \dots + a_{2n})}_{\text{G.P.}} = 5 \cdot \underbrace{(a_1 + a_3 + a_5 + \dots + a_{2n-1})}_{\text{G.P.}}$$

$$\Rightarrow (a + ar + \dots + ar^{2n-1}) = 5 \cdot (a + ar^2 + ar^4 + \dots + ar^{2n-2})$$

$$\boxed{\begin{array}{l} \text{G.P. } a_1 = a \\ \text{C.R.} = r \\ \text{No. of terms} = 2n \end{array}}$$

$$\boxed{\begin{array}{l} \text{G.P. } a_1 = a \\ \text{C.R.} = r^2 \\ \text{No. of terms} = n \end{array}}$$

$$\Rightarrow \frac{a(r^{2n} - 1)}{r - 1} = 5x \frac{a(\cancel{r^2}^n - 1)}{r^2 - 1}$$

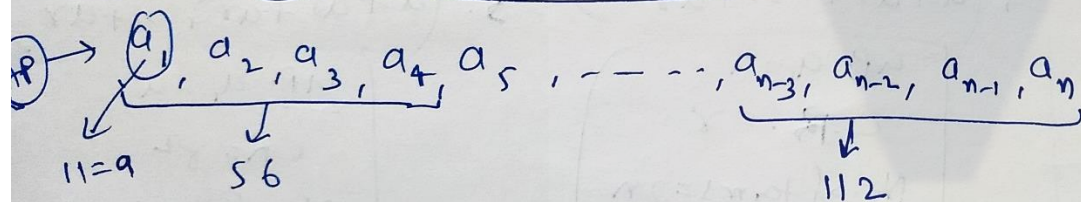
$$\Rightarrow \frac{1}{r-1} = 5 \times \frac{1}{r^2-1^2}$$

$$\Rightarrow \frac{1}{\cancel{r-1}} = \frac{5}{(\cancel{r-1})(r+1)}$$

$$\Rightarrow 1 = \frac{5}{r+1}$$

$$\Rightarrow r+1=5 \Rightarrow \boxed{r=4}$$

Q. 12 (AP) no. of terms = n = ?



ATQ. $a_1 + a_2 + a_3 + a_4 = 56$

$$\Rightarrow a + (a+d) + (a+2d) + (a+3d) = 56$$

$$\Rightarrow 4a + 6d = 56$$

$$4a + 6d = 56$$

$$\Rightarrow \boxed{a=11}$$

$$4 \times 11 + 6d = 56$$

$$\Rightarrow 44 + 6d = 56$$

$$\Rightarrow 6d = 12$$

$$\boxed{d=2}$$

ATQ.

$$a_{n-3} + a_{n-2} + a_{n-1} + a_n = 112$$

$$\Rightarrow a + (n-4) \cdot d + a + (n-3) \cdot d = 112$$

$$a + (n-2) \cdot d + a + (n-1) \cdot d$$

$$\Rightarrow 4a + d\{4n-10\} = 112$$

$$\Rightarrow 44 + 2(4n-10) = 112$$

$$\Rightarrow 44 + 8n - 20 = 112$$

$$\Rightarrow 8n = 112 - 24$$

$$\Rightarrow 8n = 88$$

$$\Rightarrow \boxed{n=11}$$

Miscellaneous Exercise (9)

Q.13

To Prove

 a, b, c, d are in G.P.

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

Given $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$

Proof: I - method:

Componendo - Dividendo

$$\frac{x}{y} = \frac{z}{w} \Rightarrow \frac{x+y}{x-y} = \frac{z+w}{z-w}$$

By applying Componendo - Dividendo

$$\frac{(a+bx) + (a-bx)}{(a+bx) - (a-bx)} = \frac{(b+cx) + (b-cx)}{(b+cx) - (b-cx)} = \frac{(c+dx) + (c-dx)}{(c+dx) - (c-dx)}$$

$$\Rightarrow \frac{2a}{2bx} = \frac{2b}{2cx} = \frac{2c}{2dx} \Rightarrow \frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

Reciprocal

$$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

II - method:

$$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$$

(1) (1) (1)

$$\Rightarrow \frac{a+bx}{a-bx} + 1 = \frac{b+cx}{b-cx} + 1 = \frac{c+dx}{c-dx} + 1$$

$$\Rightarrow \frac{a+bx+a-bx}{a-bx} = \frac{b+cx+b-cx}{b-cx} = \frac{c+dx+c-dx}{c-dx}$$

$$\Rightarrow \frac{2a}{a-bx} = \frac{2b}{b-cx} = \frac{2c}{c-dx}$$

$$\Rightarrow \frac{a-bx}{a} = \frac{b-cx}{b} = \frac{c-dx}{c}$$

(-1) (-1) (-1)

$$\Rightarrow \frac{a-bx-a}{a} = \frac{b-cx-b}{b} = \frac{c-dx-c}{c}$$

$$\Rightarrow \frac{-bx}{a} = \frac{-cx}{b} = \frac{-dx}{c} \Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

Q.14 $AP \rightarrow a, ar, ar^2, \dots, a \cdot r^{n-1}$
 $(a_1) (a_2) (a_3) \dots (a_n)$

$$S = \text{Sum} = \frac{a(r^n - 1)}{r - 1}$$

$$\begin{aligned} P = \text{Product} &= (a) \times (ar) \times (ar^2) \times \dots \times (ar^{n-1}) \\ &= a^n \times r^{1+2+3+\dots+(n-1)} \\ &= a^n \cdot r^{\frac{(n-1)}{2} \cdot (1+n-1)} \quad \text{AP} \rightarrow S_n = \frac{n}{2}(2a + (n-1)d) \\ &= a^n \cdot r^{\frac{n(n-1)}{2}} \quad \text{LHS} = \frac{n}{2}(a+l) \\ P &= a^n \cdot r^{\frac{n(n-1)}{2}} = a^n \cdot r^{\frac{n(n-1)}{2}} \end{aligned}$$

$$R = \text{Sum of Reciprocals} = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \dots + \frac{1}{ar^{n-1}}$$

First = $\frac{1}{a}$, C.R. = $\frac{\frac{1}{ar}}{\frac{1}{a}} = \frac{1}{r}$

$$\begin{aligned} R &= \frac{\frac{1}{a} \left(\left(\frac{1}{r} \right)^n - 1 \right)}{\left(\frac{1}{r} - 1 \right)} = \frac{1}{a} \cdot \frac{\left(\frac{1}{r^n} - 1 \right)}{\left(\frac{1-r}{r} \right)} \\ &= \frac{\left(\frac{1-r^n}{r^n} \right)}{a \left(\frac{1-r}{r} \right)} = \frac{(1-r^n)}{a(1-r) \cdot r^{n-1}} \end{aligned}$$

To Prove:

$$P^2 R^n = S^n$$

$$\Rightarrow \left[a^n \cdot r^{\frac{n(n-1)}{2}} \right]^2 \cdot \left[\frac{(1-r^n)}{a(1-r) \cdot r^{n-1}} \right]^n$$

$$= \left[\frac{a(r^n - 1)}{r - 1} \right]^n$$

$$\Rightarrow \frac{a^{2n-n} \cdot r^{n(n-1)} \cdot (1-r^n)^n}{(r-1)^n \cdot r^{(n-1) \cdot n}}$$

$$= \frac{a^n \cdot (r^n - 1)^n}{(r - 1)^n}$$

$$\Rightarrow \frac{a^n \cdot (r^n - 1)^n}{(r - 1)^n} = \frac{a^n \cdot (r^n - 1)^n}{(r - 1)^n} \quad \text{LHS} = \text{RHS} \quad \checkmark$$

Miscellaneous Exercise (9) (class - 11)

(15) $a = p^{\text{th}}$ term $= A + (p-1) \cdot D$
 $b = q^{\text{th}}$ term $= A + (q-1) \cdot D$
 $c = r^{\text{th}}$ term $= A + (r-1) \cdot D$

First term $= A$
 $CD = D$

To Prove: $(q-r) \cdot a + (r-p) \cdot b + (p-q) \cdot c = 0$

Proof: $(q-r) \cdot a = (q-r) \cdot [A + (p-1)D] = (q-r) \cdot A + (q-r) \cdot (p-1)D$

$(q-r) \cdot a = (q-r)A + (pq - q - rp + r)D \quad \text{--- (1)}$

Similarly $(r-p) \cdot b = (r-p) \cdot A + (rq - r - pq + p) \cdot D \quad \text{--- (2)}$

Similarly, $(p-q) \cdot c = (p-q) \cdot A + (pr - p - qr + q)D \quad \text{--- (3)}$

Add $\rightarrow +$

$\Rightarrow (q-r) \cdot a + (r-p) \cdot b + (p-q) \cdot c = (0) \cdot A + (0) \cdot D = 0 = \text{RHS.}$

Q. 16 If $a\left(\frac{1}{b} + \frac{1}{c}\right)$, $b\left(\frac{1}{c} + \frac{1}{a}\right)$, $c\left(\frac{1}{a} + \frac{1}{b}\right)$ are in AP,
then prove that a, b, c are in AP.

Given, $a\left(\frac{1}{b} + \frac{1}{c}\right)$, $b\left(\frac{1}{c} + \frac{1}{a}\right)$, $c\left(\frac{1}{a} + \frac{1}{b}\right) \rightarrow \text{AP}$

$$\Rightarrow a\left(\frac{c+b}{bc}\right), b\left(\frac{a+c}{ca}\right), c\left(\frac{b+a}{ab}\right) \rightarrow \text{AP.}$$

$$\Rightarrow \frac{ac+ab}{bc}, \frac{ab+bc}{ca}, \frac{bc+ca}{ab} \rightarrow \text{AP.}$$

Add +1 in all the terms

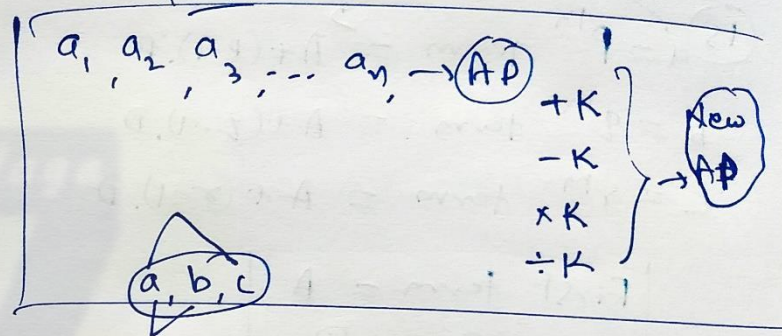
$$\Rightarrow \frac{ac+ab}{bc} + 1, \frac{ab+bc}{ca} + 1, \frac{bc+ca}{ab} + 1 \rightarrow \text{AP.}$$

$$\Rightarrow \frac{ac+ab+bc}{bc}, \frac{ab+bc+ca}{ca}, \frac{bc+ca+ab}{ab} \rightarrow \text{AP}$$

Divide $K = (ab+bc+ca)$ in all the terms

$$\Rightarrow \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab} \rightarrow \text{AP.}$$

Note:



multiply $K = abc$ in each term

$$\Rightarrow \frac{abc}{bc}, \frac{abc}{ca}, \frac{abc}{ab} \rightarrow \text{AP}$$

$$\Rightarrow a, b, c \rightarrow \text{AP.}$$

Q.17 ~~A~~ $a, b, c, d \rightarrow \text{G.P.}$ (Given)

To prove: $(a^n + b^n), (b^n + c^n), (c^n + d^n) \rightarrow \text{G.P.}$

Proof:

$$\begin{array}{ccccccc} a, & b, & c, & d & \rightarrow & \text{G.P.} \\ \downarrow & \downarrow & \downarrow & \searrow & & \\ a & ar & ar^2 & ar^3 & & \end{array}$$

$$\frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n}$$

$$\Rightarrow \frac{(ar)^n + (ar^2)^n}{(a)^n + (ar)^n} = \frac{(ar^2)^n + (ar^3)^n}{(ar)^n + (ar^2)^n}$$

$$\Rightarrow \frac{a^n \cdot r^n + a^n \cdot r^{2n}}{a^n + a^n \cdot r^n} = \frac{a^n \cdot r^{2n} + a^n \cdot r^{3n}}{a^n \cdot r^n + a^n \cdot r^{2n}}$$

$$\Rightarrow \frac{\cancel{a^n} r^n (1 + \cancel{r^n})}{\cancel{a^n} (1 + \cancel{r^n})} = \frac{\cancel{a^n} \cdot r^{2n} (1 + \cancel{r^n})}{\cancel{a^n} \cdot r^n (1 + \cancel{r^n})}$$

$$\Rightarrow r^n = \frac{r^{2n}}{r^n}$$

$$\Rightarrow r^n = r^{2n-n}$$

$$\frac{r^n}{r^n} = \frac{r^n}{r^n}$$

LHS = RHS

$$\frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n}$$

$$\Rightarrow a^n + b^n, b^n + c^n, c^n + d^n \rightarrow \underline{\underline{\text{G.P.}}}$$

Toppers Village

Miscellaneous Exercise (9) (class 11)

Q.18

$$x^2 - 3x + p = 0 \begin{cases} \rightarrow a \\ \rightarrow b \end{cases}$$

$$x^2 - 12x + q = 0 \begin{cases} \rightarrow c \\ \rightarrow d \end{cases}$$

To Prove

$$(q+p):(q-p) = 17:15$$

$$\begin{array}{c} a, b, c, d \rightarrow q, p. \\ \downarrow \quad \downarrow \quad \downarrow \quad \searrow \\ a \quad ar \quad ar^2 \quad ar^3 \\ \hline \end{array}$$

$$\begin{cases} x^2 - 3x + p = 0 \begin{cases} \rightarrow a \\ \rightarrow b \end{cases} \\ a+b=3 \checkmark \\ ab=p \checkmark \end{cases}$$

$$\begin{cases} x^2 - 12x + q = 0 \begin{cases} \rightarrow c \\ \rightarrow d \end{cases} \\ c+d=12 \checkmark \\ cd=q \checkmark \end{cases}$$

$$a+b=3 \Rightarrow a+ar=3 \Rightarrow a(1+r)=3 \quad \text{--- (1)}$$

$$c+d=12 \Rightarrow ar^2+ar^3=12 \Rightarrow ar^2(1+r)=12 \quad \text{--- (2)}$$

Divide.

$$\frac{1}{r^2} = \frac{3}{12} \Rightarrow \frac{1}{r^2} = \frac{1}{4} \Rightarrow r^2 = 4 \Rightarrow \boxed{r = \pm 2}$$

$$\because a+b=3 \Rightarrow a+ar=3 \Rightarrow \boxed{a(1+r)=3} \Rightarrow \boxed{a = \frac{3}{1+r}}$$

$$\begin{array}{l} \boxed{r = +2} \\ \rightarrow a = \frac{3}{1+r} = \frac{3}{1+2} \\ \rightarrow \boxed{a = 1} \end{array}$$

$$\begin{array}{l} \boxed{r = -2} \\ \rightarrow a = \frac{3}{1+r} = \frac{3}{1-2} = \frac{3}{-1} \\ \rightarrow \boxed{a = -3} \end{array}$$

$$\text{LHS} = (q+p):(q-p)$$

$$\text{RHS} = 17:15$$

$$p = ab \quad q = cd$$

$$p = ar \cdot ar \quad q = (ar^2) \cdot (ar^3)$$

$$p = a^2 r \quad q = a^2 r^5$$

$$\begin{aligned} r^2 &= 4 \\ \Rightarrow r^4 &= 16 \end{aligned}$$

$$\text{LHS} = (a^2 r^5 + a^2 r) : (a^2 r^5 - a^2 r)$$

$$= \frac{\cancel{a^2} r (r^4 + 1)}{\cancel{a^2} r (r^4 - 1)} = \frac{r^4 + 1}{r^4 - 1} = \frac{16 + 1}{16 - 1}$$

$$= \frac{17}{15} = 17:15 = \text{RHS.}$$

$$(q+p):(q-p) = 17:15$$

[Q.19] Two positive No. = a, b

$$\text{AM} = \frac{a+b}{2}$$

$$\text{GM} = \sqrt{ab}$$

$$\frac{\text{AM}}{\text{GM}} = \frac{m}{n}$$

To Prove:

$$a:b = (m + \sqrt{m^2 - n^2}) : (m - \sqrt{m^2 - n^2})$$

$$\frac{a}{b} = \frac{(m + \sqrt{m^2 - n^2})}{m - \sqrt{m^2 - n^2}} \times \frac{(m + \sqrt{m^2 - n^2})}{m + \sqrt{m^2 - n^2}}$$

$$\frac{a}{b} = \frac{m^2 + m^2 - n^2 + 2m\sqrt{m^2 - n^2}}{m^2 - (m^2 - n^2)}$$

$$\frac{a}{b} = \frac{2m^2 - n^2 + 2m\sqrt{m^2 - n^2}}{n^2}$$

~~Given~~ $\frac{A}{a} = \frac{m}{n}$

$$\Rightarrow \frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{m}{n}$$

$$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$$

Square

$$\Rightarrow \frac{(a+b)^2}{(2\sqrt{ab})^2} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{a^2+b^2+2ab}{4 \cdot ab} = \frac{m^2}{n^2}$$

$$\Rightarrow \left(\frac{a}{b}\right) + \frac{b}{a} + 2 = \frac{4m^2}{n^2}$$

$$\frac{a}{b} = x \text{ (let } x=?)$$

$$\Rightarrow \frac{x}{1} + \frac{1}{x} + \frac{2}{1} = \frac{4m^2}{n^2}$$

$$\Rightarrow \frac{x^2 + 1 + 2x}{x} = \frac{4m^2}{n^2}$$

$$\Rightarrow \underbrace{x^2}_{n^2 x^2} + \underbrace{1}_{n^2} + \underbrace{2x}_{2x \cdot n^2} = \frac{x \cdot 4m^2}{n^2}$$

$$\Rightarrow n^2 x^2 + 2x \cdot n^2 - x \cdot 4m^2 + n^2 = 0$$

$$\Rightarrow \underline{n^2 \cdot x^2} + x(\underline{2n^2 - 4m^2}) + \underline{n^2} = 0$$

$$x = \frac{-2n^2 + 4m^2 \pm \sqrt{(2n^2 - 4m^2)^2 - 4 \cdot n^4}}{2n^2}$$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

16 m²

$$x = \frac{-2n^2 + 4m^2 \pm \sqrt{4n^4 + 16m^4 - 16n^2m^2 - 4n^4}}{2n^2}$$

$$x = \frac{-2n^2 + 4m^2 \pm 4m \sqrt{m^2 - n^2}}{2n^2}$$

$$\frac{a}{b} = x = \frac{-n^2 + 2m^2 \pm 2m \sqrt{m^2 - n^2}}{n^2}$$

Q.20

$$a, b, c \rightarrow AP \rightarrow \boxed{b = \frac{a+c}{2}} \quad (1)$$

$$b, c, d \rightarrow AP \rightarrow \boxed{c^2 = bd} \quad (2)$$

$$\frac{1}{c}, \frac{1}{d}, \frac{1}{e} \rightarrow AP$$

$$\rightarrow \frac{1}{d} = \frac{\frac{1}{c} + \frac{1}{e}}{2}$$

$$\Rightarrow \frac{1}{d} = \frac{\frac{e+c}{ce}}{2}$$

$$\Rightarrow \frac{1}{d} = \frac{e+c}{2ce}$$

$$\Rightarrow \boxed{d = \frac{2ce}{e+c}} \quad (3)$$

By eqn (2): $c^2 = bd$

$$\Rightarrow c^2 = \left(\frac{a+c}{2}\right) \cdot \left(\frac{2ce}{e+c}\right)$$

$$\Rightarrow \cancel{c^2 = ace +}$$

to prove $\boxed{a, c, e \rightarrow AP.}$

$$\rightarrow \boxed{c^2 = ae}$$

$$\Rightarrow c = \frac{(a+c) \cdot (e)}{(e+c)}$$

$$\Rightarrow \cancel{ec + c^2 = ae + ce}$$

$$\Rightarrow \boxed{c^2 = ae}$$

$\therefore a, c, e$ are in AP.

class - 11 - Maths. Miscellaneous Exercise (9)

Q.21 Find Sum of n -terms?

(i) $5 + 55 + 555 + \dots$

Sum of n -terms $= S_n = \underline{5} + \underline{55} + \underline{555} + \dots$ n -terms

$\Rightarrow S_n = 5 \cdot (1 + 11 + 111 + \dots$ $\xleftarrow{\text{self}} \dots$ n -terms $) \times \frac{9}{9}$

$= \frac{5}{9} \cdot (\underline{9} + \underline{99} + \underline{999} + \dots$ n -terms $)$

$= \frac{5}{9} \cdot ([\underline{10-1}] + [\underline{100-1}] + [\underline{1000-1}] + \dots$ n -terms $)$

$= \frac{5}{9} \left(\frac{[10 + 100 + 1000 + \dots$ n -terms $] - [1 + 1 + 1 \dots$ n -terms $]}{a=10, r=10 \text{ G.P. } \rightarrow S = \frac{a(r^n-1)}{r-1}} \right)$

$= \left(\frac{5}{9} \right) \left\{ \left[\frac{10(10^n-1)}{10-1} \right] - [n] \right\}$

$= \frac{50}{81} \cdot (10^n-1) - \frac{5n}{9} \checkmark$

$$(ii) \quad 0.6 + 0.66 + 0.666 + \dots \text{ } n\text{-terms}$$

$$S_n = 0.6 + 0.66 + 0.666 + \dots \text{ } n\text{-terms}$$

$$S_n = 6 \cdot \left\{ 0.1 + 0.11 + 0.111 + \dots \text{ } n\text{-terms} \right\} \times \frac{9}{9}$$

$$= \frac{2}{3} \cdot \left\{ \frac{0.9}{1} + 0.99 + 0.999 + \dots \text{ } n\text{-terms} \right\}$$

$$= \frac{2}{3} \cdot \left\{ \frac{1-0.1}{1-0.1} + \frac{1-0.01}{1-0.01} + \frac{1-0.001}{1-0.001} + \dots \text{ } n\text{-terms} \right\}$$

$$= \frac{2}{3} \left\{ (1+1+1+\dots \text{ } n\text{-terms}) - (0.1+0.01+0.001+\dots \text{ } n\text{-terms}) \right\}$$

$$= \frac{2}{3} \left\{ n - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots \text{ } n\text{-terms} \right) \right\}$$

$$= \frac{2}{3} \left\{ n - \frac{\frac{1}{10} \left(1 - \left(\frac{1}{10} \right)^n \right)}{\left(1 - \frac{1}{10} \right)} \right\}$$

$G.P. \quad a = \frac{1}{10}, r = \frac{1}{10} \quad S = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}$
 $r < 1$

$$= \frac{2}{3} \left\{ n - \frac{\frac{1}{10} \cdot \left(1 - \frac{1}{10^n} \right)}{\frac{9}{10}} \right\} = \frac{2n}{3} - \frac{2 \left(1 - \frac{1}{10^n} \right)}{27}$$

Q.22 20th term of series

$$\frac{2 \times 4}{a_1} + \frac{4 \times 6}{a_2} + \frac{6 \times 8}{a_3} + \dots + n \text{ terms } a_n$$

$a_{20} = ?$

$$\frac{2}{2 \times 1}, \frac{4}{2 \times 2}, \frac{6}{2 \times 3}, \dots$$

Similarly n^{th} term = $(2n) \times (2n+2) = a_n$

$n=20$
 $a_{20} = (2 \times 20) \times (2 \times 20 + 2)$

$a_{20} = 40 \times 42$ ✓

$a_{20} = 1680$ ✓

Q.23

$? = S_n = \overbrace{3+7+13+21+31+\dots+a_n}^{n\text{-terms}}$
 not in A.P.
 Differences: 4, 6, 8, 10
 AP ✓

$a_n = ? \rightarrow S_n = ?$

$S_n = 3 + 7 + 13 + 21 + 31 + \dots + a_{n-1} + a_n$

$- S_n = \quad \quad \quad 3 + 7 + 13 + 21 + \dots + a_{n-2} + a_{n-1} + a_n$

$0 = 3 + 4 + 6 + 8 + 10 + \dots + () + () - a_n$

AP $\rightarrow$ Sum?

$\Rightarrow a_n = 3 + \frac{(n-1)}{2} \cdot [2 \times 4 + (n-2) \cdot 2]$
 no. of terms = $(n-1)$
 $a = 4$
 $d = 2$

$\Rightarrow a_n = 3 + \frac{(n-1)}{2} \cdot (4 + n - 2)$

$\Rightarrow a_n = 3 + (n-1) \cdot (n+2) = 3 + (n^2 + n - 2)$

$a_n = n^2 + n + 1$

$$a_n = n^2 + n + 1$$

Some Special Series :

$$S_n = a_1 + a_2 + a_3 + \dots + a_n$$

$$S_n = \sum_{k=1}^n a_k$$

$$S_n = \sum_{k=1}^n (k^2 + k + 1)$$

$$S_n = \left(\sum_{k=1}^n k^2 \right) + \left(\sum_{k=1}^n k \right) + \left(\sum_{k=1}^n 1 \right)$$

$\rightarrow 1 + 1 + 1 + \dots + 1$
 $n\text{-times}$

$$S_n = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + n$$

$$S_n = n \left\{ \frac{2n^2 + 3n + 1}{6} + \frac{n+1}{2} + \frac{1}{1} \right\}$$

$$S_n = n \left\{ \frac{2n^2 + 3n + 1 + 3n + 3 + 6}{6} \right\}$$

$$S_n = n \left\{ \frac{n^2 + 3n + 5}{3} \right\}$$

$$S_n = \frac{n(n^2 + 3n + 5)}{3}$$

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Miscellaneous Exercise - chapter (9)

Q. 24

$$\text{Sum of first } n\text{-natural No.} = S_1 = 1+2+3+\dots+n = \frac{n(n+1)}{2} \quad \text{2.} = \sum_{k=1}^n k$$

$$\text{Sum of their ~~cubes~~ squares.} = S_2 = 1^2+2^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6} = \sum_{k=1}^n k^2$$

$$\text{Sum of their cubes.} = S_3 = 1^3+2^3+\dots+n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \sum_{k=1}^n k^3$$

To Prove $9S_2^2 = S_3(1+8S_1)$

$$\text{RHS} = (S_3)(1+8S_1)$$

$$= \left(\frac{n(n+1)}{2}\right)^2 \cdot \left(1 + \frac{4}{\cancel{8}} \cdot \frac{n(n+1)}{\cancel{2}}\right)$$

$$= \left[\frac{n(n+1)}{2}\right]^2 \cdot \left[1 + \underbrace{4n^2 + 4n}_{(2n+1)^2}\right]$$

$$= \left[\frac{n(n+1)}{2}\right]^2 \cdot (2n+1)^2 = \left[\frac{n(n+1)(2n+1)}{2}\right]^2$$

$$\text{LHS} = 9(S_2)^2$$
$$= 9 \cdot \left[\frac{n(n+1)(2n+1)}{6}\right]^2$$

$$= \left[\frac{n(n+1)(2n+1)}{2}\right]^2$$

$$= \text{RHS.}$$

Q.25 Sum of n -terms $= ? = S_n$

$$S_n = \frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots + \boxed{a_n}$$

$\uparrow$ $\uparrow$ $\uparrow$
 a_1 a_2 a_3

odd $\begin{cases} 2n-1 \\ 2n+1 \end{cases}$ $\begin{matrix} n=3 \\ =5 \\ =7 \end{matrix}$

n^{th} term $= a_n =$

$$\frac{1^3+2^3+3^3+\dots+n^3}{1+3+5+\dots+(2n-1)} \Rightarrow a_n = \frac{\left[\frac{n(n+1)}{2} \right]^2}{\frac{n}{2} \left[\frac{1}{2} + (n-1) \cdot \frac{1}{2} \right]}$$

$\downarrow$ Sum $\rightarrow$ AP $\begin{matrix} \text{no. of terms} \\ = n \\ a=1 \\ d=2 \end{matrix}$

$$a_n = \frac{\frac{n^2 \cdot (n+1)^2}{2^2}}{n \left(\frac{1}{2} + (n-1) \cdot \frac{1}{2} \right)} = \frac{\frac{n^2 \cdot (n+1)^2}{2^2}}{\frac{n^2}{2}} = \frac{(n+1)^2}{4} = \frac{n^2+2n+1}{4} = \frac{n^2}{4} + \frac{n}{2} + \frac{1}{4}$$

$$\boxed{a_n = \frac{n^2}{4} + \frac{n}{2} + \frac{1}{4}}$$

$$a_n = \frac{n^2}{4} + \frac{n}{2} + \frac{1}{4}$$

$$S_n = \sum_{k=1}^n a_k$$

$$S_n = \sum_{k=1}^n \left(\frac{k^2}{4} + \frac{k}{2} + \frac{1}{4} \right)$$

$$S_n = \sum \frac{k^2}{4} + \sum \frac{k}{2} + \sum \left(\frac{1}{4} \times 1 \right)$$

$$S_n = \frac{1}{4} \left(\sum_{k=1}^n k^2 \right) + \frac{1}{2} \left(\sum_{k=1}^n k \right) + \frac{1}{4} \left(\sum_{k=1}^n 1 \right)$$

$$S_n = \frac{1}{4} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \cdot \left(\frac{n(n+1)}{2} \right) + \frac{1}{4}(n)$$

Q.26 Show that $\frac{1 \cdot 2^2 + 2 \cdot 3^2 + \dots + n \cdot (n+1)^2}{1^2 \times 2 + 2^2 \cdot 3 + \dots + n^2 \cdot (n+1)} = \frac{3n+5}{3n+1}$

Numerator

$$S_n^N = \underbrace{1 \cdot 2^2}_{a_1} + \underbrace{2 \cdot 3^2}_{a_2} + \dots + \underbrace{n \cdot (n+1)^2}_{a_n}$$

$$a_n = n(n+1)^2 = n(n^2 + 2n + 1)$$

$$(a_n) = n^3 + 2n^2 + n \checkmark$$

$$S_n^N = \sum_{k=1}^n (a_k) = \sum_{k=1}^n (k^3 + 2k^2 + k)$$

$$= \left(\sum_{k=1}^n k^3 \right) + 2 \left(\sum_{k=1}^n k^2 \right) + \left(\sum_{k=1}^n k \right)$$

$$= \left[\frac{n(n+1)}{2} \right]^2 + 2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$n(n+1) = (n^2 + n)$$

$$= n(n+1) \left\{ \frac{n(n+1)}{4} + \frac{2n+1}{3} + \frac{1}{2} \right\}$$

$$= n(n+1) \cdot \left\{ \frac{3n^2 + 3n + 8n + 4 + 6}{12} \right\}$$

$$= \frac{n(n+1) (3n^2 + 11n + 10)}{12}$$

$$= n(n+1) \cdot \frac{(3n^2 + 6n + 5n + 10)}{12}$$

$$S_n^N = \frac{n(n+1) \cdot (3n+5) \cdot (n+2)}{12} \checkmark$$

$$S_n^D = \frac{1^2 \cdot 2}{\uparrow a_1} + \frac{2^2 \cdot 3}{\uparrow a_2} + \dots + \frac{n^2 \cdot (n+1)}{\uparrow a_n}$$

$$a_n = n^2 \cdot (n+1)$$

$$a_n = n^3 + n^2$$

$$S_n^D = \sum_{k=1}^n a_k = \sum_{k=1}^n k^3 + k^2$$

$$S_n^D = \left(\sum_{k=1}^n k^3 \right) + \sum_{k=1}^n k^2$$

$$= \left[\frac{n(n+1)}{2} \right]^2 + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{4} \left\{ \frac{n(n+1)}{4} + \frac{2n+1}{6} \right\}$$

$$= \frac{n(n+1)(3n^2 + 3n + 4n + 2)}{12}$$

$$= \frac{n(n+1)(3n^2 + 7n + 2)}{12}$$

$$= \frac{n(n+1)(3n^2 + 6n + n + 2)}{12}$$

$$S_n^D = \frac{n(n+1) \cdot (3n+1)(n+2)}{12}$$

$$\frac{S_n^N}{S_n^D} = \frac{\cancel{n(n+1)}(3n+5)(\cancel{n+2})}{\cancel{n(n+1)}(3n+1)(\cancel{n+2})}$$

$$= \frac{3n+5}{3n+1} = \text{RHS.}$$

Q. 28

original
Total Price = ₹22000

₹4000 cash

₹18000 Installments

(₹1000 × 18 installments)

1 st →		1000 + 10% of 18000
2 nd →		1000 + 10% of 17000
3 rd →		1000 + 10% of 16000
		⋮
		⋮
		⋮
18 th →		1000 + 10% of 1000
	+	

$$\begin{aligned} \text{Total cost of} \\ \text{Scooter} &= \overbrace{4000 + 18000} \\ &\quad + 17100 \\ &= ₹39100 \end{aligned}$$

$$\text{Total} \quad 1000 \times 18 + \frac{10}{100} \text{ of } (18000 + 17000 + \dots + 1000)$$

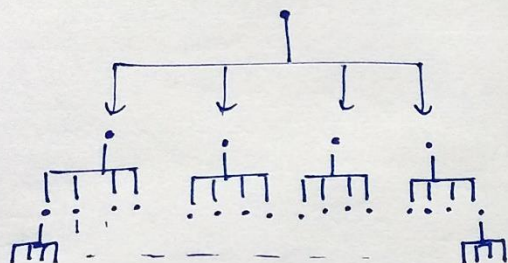
$$= 18000 + \frac{10}{100} \times \frac{n}{2} (a + l) \quad (\text{A.P. } n=18)$$

$$= 18000 + \frac{10}{100} \times \frac{18}{2} (1000 + 18000)$$

$$= 18000 + \frac{1}{10} \times 9 \times 19000 = 18000 + 17100$$

Q.29

₹ 0.50 = 50 paise for one letter



$$\left. \begin{array}{l} 1^{st} \text{ set} \end{array} \right\} \rightarrow \text{Cost}_1 \rightarrow 1 \times 4 \times 0.50 = ₹ 2 = (a_1)$$

$$\left. \begin{array}{l} 2^{nd} \text{ set} \end{array} \right\} \rightarrow \text{Cost}_2 \rightarrow 4 \times 4 \times 0.50 = ₹ 8 = (a_2)$$

$$\left. \begin{array}{l} 3^{rd} \text{ set} \end{array} \right\} \rightarrow \text{Cost}_3 \rightarrow 4 \times 4 \times 4 \times 0.50 = ₹ 32 = (a_3)$$

$$\vdots$$

$$\left. \begin{array}{l} 8^{th} \text{ set} \end{array} \right\} \rightarrow \text{Cost}_8 \rightarrow \dots = (a_8)$$

total cost

Series = $(₹ 2) + (₹ 8) + (₹ 32) + \dots + (a_8) \rightarrow \text{G.P.}$

$$\text{Sum of G.P.} = \frac{a(r^n - 1)}{r - 1} \Rightarrow \begin{cases} a = 2 \\ r = 4 \\ n = 8 \end{cases}$$

$$\text{total cost} = \frac{2(4^8 - 1)}{4 - 1} = 2 \times \frac{21845}{3}$$

$$= ₹ 43690$$

Rough

$$48 = 2^{16}$$

$$2^{12} = 4096$$

$$2^{16} = 2^{12} \times 2^4$$

$$= 4096 \times 16$$

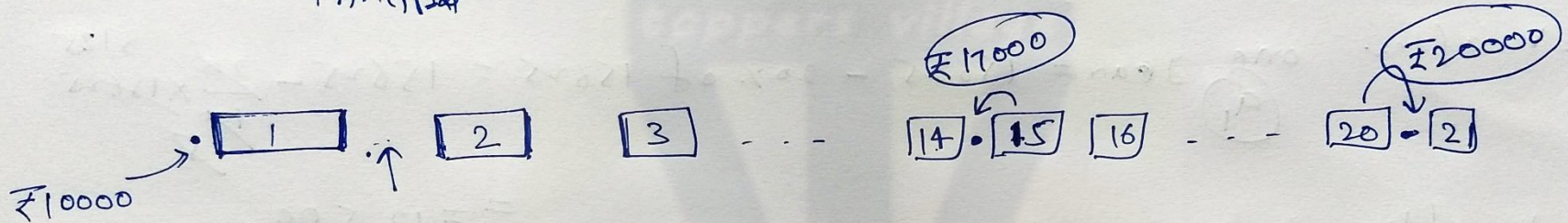
$$= 65536$$

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Class-11-Maths Miscellaneous Exercise (9)

Q.30 Deposited ₹10,000 | Rate = 5% (SI) | Amount = Principal + Interest

Principal



$$\text{Amount} = \text{Principal} + \text{SI}$$

$$= 10000 + 5\% \text{ of } 10000$$

$$= 10000 + \frac{5}{100} \times 10000$$

$$= 10000 + 500$$

$$= ₹10500$$

$$\text{After one year} \rightarrow \text{Amount}_1 = 10000 + 500$$

$$\text{After two years} \rightarrow A_2 = 10000 + 500 + 500$$

$$\text{After 3-years} \rightarrow A_3 = 10000 + 500 \times 3$$

⋮

$$\text{After 14-years} \rightarrow A_{14} = 10000 + 500 \times 14$$

Amount in 15th year

$$= 10000 + 7000 = 17000$$

$$\text{After 20-years} \rightarrow A_{20} = 10000 + 500 \times 20 = 10000 + 10000$$

[Q.3]

Present cost = ₹ 15625

Rate = -20%.

1 2 3 4 5
15625 12500 10000

Estimated value at the end of 5 years = ? = 5120

E.V.

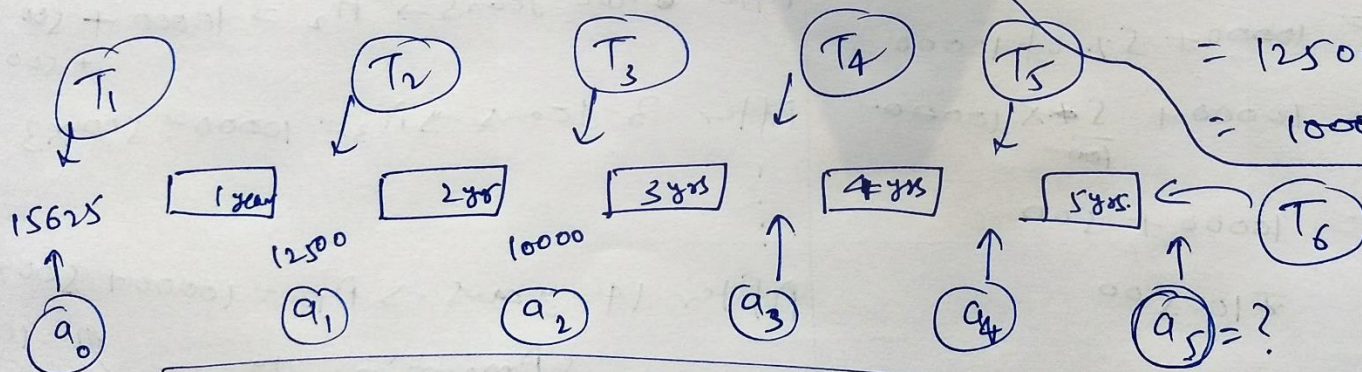
EV after one year = $15625 - 20\% \text{ of } 15625 = 15625 - \frac{20}{100} \times 15625$

E.V. after 2 years = $12500 - 20\% \text{ of } 12500 = 12500 - \frac{20}{100} \times 12500$

= ₹ 12500

= $12500 - 2500$

= 10000



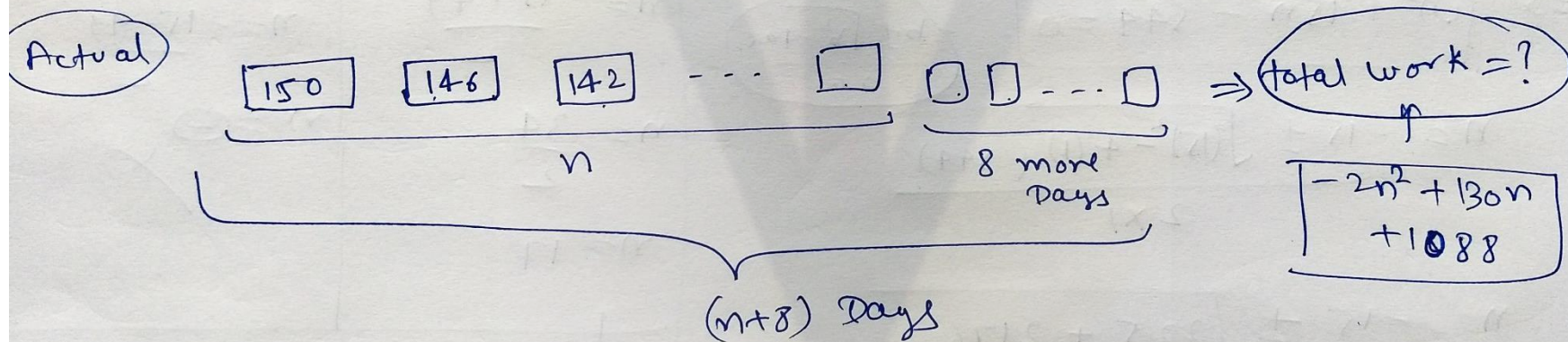
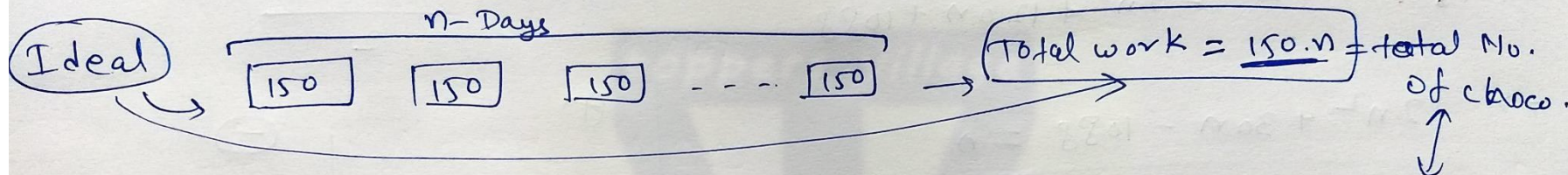
$$\frac{12500}{15625} = \frac{4}{5} = \frac{10000}{12500}$$

G.P. $\rightarrow a = 15625$
 $r = \frac{4}{5}$

$$\begin{aligned} a_5 &= T_6 \\ &= a \cdot r^{n-1} \\ &= 15625 \cdot \left(\frac{4}{5}\right)^5 \\ &= 5120 \end{aligned}$$

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Q.32



In ~~the~~ actual case, total work = $150 + 146 + 142 + \dots$

Sum

(A.P.) $a = (150)$
 $d = (-4)$
no. of terms = $(n+8)$

$$= \frac{(n+8)}{2} \left[2 \times 150 + (n+7) \cdot (-4) \right]$$

$$= (n+8) (150 - 2n - 14) = (n+8) (136 - 2n) = -2n^2 + 130n + 1088$$

$$\begin{array}{c} \text{total work} \\ \swarrow \quad \searrow \\ \text{Ideal} \quad = \quad \text{actual} \end{array}$$

$$\Rightarrow 150.n = -2n^2 + 120n + 1088$$

$$\Rightarrow 2n^2 + 30n - 1088 = 0$$

$$\Rightarrow \boxed{1.n^2 + 15n - 544 = 0} \quad \left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$$

$$n = \frac{-15 \pm \sqrt{(15)^2 - 4.(1).(-544)}}{2 \times 1}$$

$$n = \frac{-15 \pm \sqrt{225 + 2176}}{2}$$

$$n = \frac{-15 \pm \sqrt{2401}}{2}$$

$$\sqrt{2401} = \boxed{49}$$

$$n = \frac{-15 \pm 49}{2}$$

⊕

$$n = \frac{-15 + 49}{2}$$

$$n = \frac{34}{2}$$

$$n = 17$$

⊖

$$n = \frac{-15 - 49}{2}$$

$$n = \ominus$$

↓

no. of Days in which the work was completed. actual

$$= (n+8)$$

$$= 17+8$$

$$= 25 \text{ Days}$$

All Useful Links:

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** www.toppersvillage.com
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** www.facebook.com/ToppersVillage/
- **Facebook Group:** <https://www.facebook.com/groups/ToppersVillage>